Off-Grid Localization in MIMO Radars Using Sparsity

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***Abstract*—In this letter, we propose a new accurate approach for target localization in multiple-input multiple-output (MIMO) radars, which exploits the sparse spatial distribution of targets to reduce the sampling rate. We express the received signal of a MIMO radar in terms of the deviations of target parameters from the grid points in the form of a block sparse signal using the expansion around all the neighbor points. Applying a block sparse recovery method, we can estimate both the grid-point locations of targets and these deviations. The proposed approach can yield more accurate localization with higher detection probability compared with its counterparts. Moreover, the proposed approach can reduce the computational complexity.**

***Index Terms*—Compressive sensing (CS), multiple-input multiple-output (MIMO) radar, off-grid localization, random sampling, sparse recovery.**

I. Introduction

MULTIPLE-INPUT multiple-output (MIMO) radars have multiple transmitters, multiple receivers, and a common processing center called fusion center [1]. In MIMO radars, a large amount of data should be sent from the receivers to the fusion center. Hence, the cost can be significantly reduced by reducing the sampling rate at each receiver using compressive sensing (CS) [2].

CS is a sampling method allowing a sampling rate less than the Nyquist rate for sparse signals. We define a vector $\mathbf{z}$ as K-sparse in a basis $\mathbf{H}$ if we can write $\mathbf{z} = \mathbf{H}\mathbf{s}$, where $\mathbf{s}$ has only K nonzero entries. The aim in the CS is to take a number of linear combinations of the input that is less than the input size. In other words, we take $\mathbf{y} = \mathbf{\Phi}\mathbf{z} = \mathbf{\Theta}\mathbf{s}$ as our measurement samples, where $\mathbf{\Phi} \in C^{M\times Q}$ is a measurement matrix with $M \ll Q$. Matrix $\mathbf{\Theta} = \mathbf{H}\mathbf{\Phi}$ is called sensing matrix. Random sampling is a spatial case of CS in which some entries of $\mathbf{z}$ are selected randomly instead of all its entries [3]–[7]. The sparse recovery methods allow precise recovery of the sparse signal from $\mathbf{y}$ if $\mathbf{\Theta}$ satisfies certain conditions [2], [8], [9].

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In MIMO radars, the received signal is a sparse signal due to the spatial sparsity of targets. Thus, by employing CS we can reduce the sampling rate. At the fusion center, the received signal is recovered and target parameters are estimated using a sparse recovery method. Several sparse recovery methods have been employed in these radars for specific situations and shown to outperform their competitors in terms of target parameter estimation using fewer samples [10], [11]. The MIMO radars that employ CS can also be improved in various ways, such as optimizing the measurement matrix [11], [12], designing the transmit waveforms [12]–[17], and using appropriate sparse recovery methods [6], [18]–[20].

To employ CS and sparse recovery methods, the space of parameters is discretized into a number of so-called grid points or cells. The approach in most literature, called on-grid approach, is to approximate the target parameters to the nearest grid-points [6], [10]–[24]. We call this approximation on-grid approximation. This approach results in an error in estimating target parameters, which is called off-grid error. One way to reduce the off-grid error is to reduce the distance between the grid points. Unfortunately, this requires a substantial increase in the size of the sensing matrix, which not only imposes extra computational cost but also results in substantial performance degradation as the coherence of this matrix increases [25], [26]. In [27]–[29], some postprocessing methods are proposed to refine the output of the on-grid algorithms around the detected target positions. These methods attempt to partially reduce the off-grid errors. In [30], the grid is alternatively refined after a sparse recovery, whereas in [31], different grids are considered and the best one is chosen. Instead of on-grid approximation, we can use off-grid ones in which the deviations of the target parameters from the grid points are also considered in approximating the received signal. One approach is to approximate the received signal of a MIMO radar from a target in term of the deviation of the target parameter from the nearest grid-point using linear Taylor expansion. In [32]–[38], this off-grid approach is used and some modified algorithms are proposed for the sparse recovery. However, linear Taylor expansion is not always the best choice.

In this letter, we propose to approximate the received signal from a target in a MIMO radar around all its neighbor grid-points and formulate a block sparse problem that allows to estimate both the target parameters and the deviation errors from the grid points. Hence, the model easily allows the use of any existing block sparse recovery algorithm in order to more accurately estimate the target parameters. Our simulation results reveal that the proposed approach not only can yield more accurate localization but is also superior than the traditional on-grid approach, the postprocessing remedies, and the Taylor expansion approach in detection ability. Furthermore, it can reduce the computational cost.

This letter is organized as follows. In Section II, the received signal of a MIMO radar is modeled and approximated using the traditional on-grid approximation and the off-grid approximations (Taylor approximation and a new proposed approximation). Section III is allocated to utilizing CS in MIMO radars, and in Section IV, we propose to employ block sparse recovery methods for accurate localization in MIMO radars. The simulation results are discussed in Section V, and Section VI concludes this letter.

II. MIMO Radar Signal Model

We consider a MIMO radar with N_r receivers and N_t transmitters all located on a line with the distances of d_m (for $m = 1, \ldots, N_t$) and $\overline{d}_n$ (for $n = 1, \ldots, N_r$) from the origin, respectively. For simplicity, we consider that targets are all stationary and are all on a two-dimensional (2-D) plane that contains all antennas. Note that our results can be easily extended to moving targets and a 3-D environment. We assume there are K targets in the search area, and (r_k, θ_k) denotes the polar coordinate of the kth target, where r_k is its distances from the origin and θ_k is its angle. Let us consider that the MIMO radar is colocated [1], [39], i.e., all antennas are located in a small area ($r_k \gg d_m$ and $r_k \gg \overline{d}_n$). Thus, all transmitter–receiver pairs view the targets from almost the same angles. The signal received by the nth receiver can be written as

$$z_n(t) = \sum_{m=1}^{N_t} \sum_{k=1}^{K} \sigma_k x_m(t - \tau_{k,m,n}) e^{j\omega_m(t-\tau_{k,m,n})} + e_n(t) \quad (1)$$

where $\omega_m = 2\pi f_m$, $x_m(t)$ is the transmit baseband signal from the mth transmitter, and f_m is the corresponding carrier frequency. The delay $\tau_{k,m,n} = (R_k^m + \overline{R}_k^n)/c$ is from the mth transmitter to the kth target and back to the nth receiver; R_k^m and $\overline{R}_k^n$ are the distances between the kth target and the mth transmitter and between the kth target and nth receiver, respectively; σ_k is the target attenuation coefficient of the kth target; c is the speed of light; and $e_n(t)$ is the additive noise. We can approximate $R_k^m \simeq r_k - d_m \eta_k$ and $\overline{R}_k^n \simeq r_k - \overline{d}_n \eta_k$, where $\eta_k = \cos(\theta_k)$. Substituting $x_m(t - \tau_{k,m,n}) \simeq x_m(t - \frac{2r_k}{c})$ into (1), we get

$$z_n(t) \simeq \sum_{m=1}^{N_t} \sum_{k=1}^{K} \sigma_k x_m\left(t - \frac{2r_k}{c}\right) \times e^{\frac{-j2\omega_m r_k}{c}} a_{m,d_m,\eta_k} a_{m,\overline{d}_n,\eta_k} + e_n(t) \quad (2)$$

where $a_{m,d,\eta_k} = \exp(\frac{j\omega_m \eta_k d}{c})$.

For the estimation process, let us assume that each transmitter sends P pulses with the pulse repetition interval (PRI) of T seconds. We assume that the transmitted signals are designed such that the received waveforms from different transmitters remain almost orthogonal such that they could be accurately separated at the receivers. For simplicity, we consider the case in which we wish to estimate the direction of arrivals (DOA), θ_k for $k = 1, \ldots, K$, or η_k for $k = 1, \ldots, K$. By denoting $z_{n,m}(t)$ as the separated signal from $z_n(t)$, the samples of $z_{n,m}(t)$ can be written as

$$z_{n,m}(t_{p,q}) = \sum_{k=1}^{K} \beta_k h_{\eta_k,p,q,m,n} + e_{n,m}(t_{p,q}) \quad (3)$$

where

$$h_{\eta_k,p,q,m,n} = a_{m,d_m,\eta_k} a_{m,\overline{d}_n,\eta_k} \quad (4)$$

and $\beta_k = \sigma_k x_m(t_{p,q} - \frac{2r_k}{c}) e^{\frac{-j2\omega_m r_k}{c}}$. Furthermore, $t_{p,q} = (p-1)T + T_0 + qT_s$ are the sampling time instances for $q = 0, 1, \ldots, Q-1$ and $p = 0, 1, \ldots, P-1$; T_0 is the initial sampling point, and $e_{n,m}(t_{p,q})$ is the separated additive noise sampled at $t_{p,q}$.

A. On-Grid Approximation

Here, we may approximate (3) for the on-grid processing methods following almost the same procedure as in [11], [14], and [16]. We choose an L-length sampling set in the space of unknown parameter such as $\mathcal{S} = \{\overline{\eta}_1, \overline{\eta}_2, \ldots, \overline{\eta}_L\}$ so that for each η_k we can find a neighbor point $\overline{\eta}_{\ell(k)}$ in $\mathcal{S}$ to have

$$h_{\eta_k,p,q,m,n} \simeq h_{\overline{\eta}_{\ell(k)},p,q,m,n}. \quad (5)$$

Furthermore, we define the sparse vector $\mathbf{s}$ as follows:

$$s_l = \begin{cases} \beta_k & \text{if } l = \ell(k), \\ 0 & \text{otherwise}, \end{cases} \qquad \mathbf{s} = [s_1, s_2, \ldots, s_L]^T. \quad (6)$$

Using (5) and (6), we can rewrite (3) for $q = 0, 1, \ldots, Q-1$ and $p = 1, 2, \ldots, P$ as

$$z_{n,m}(t_{p,q}) = \sum_{l=1}^{L} s_l h_{\overline{\eta}_l,p,q,m,n} + e'_{n,m}(t_{p,q}) \quad (7)$$

where $e'_{n,m}(t_{p,q})$ includes the additive noise $e_{n,m}(t_{p,q})$ and the approximation errors in (5). The nth received signal is separated using different transmit waveforms and sampled. Let $\mathbf{z}_{n,m,p} = [z_{n,m}(t_{p,0}), z_{n,m}(t_{p,1}), \ldots, z_{n,m}(t_{p,Q-1})]^T$ denotes the concatenated sample vector at the nth receiver from the mth transmitter and the pth pulse. Concatenating all pulses at the nth receiver from the mth transmitter, we denote $\mathbf{z}_{n,m} = \text{vec}([\mathbf{z}_{n,m,1}, \mathbf{z}_{n,m,2}, \ldots, \mathbf{z}_{n,m,P}])$. The matrix $\mathbf{z}_n = [\mathbf{z}_{n,1}, \mathbf{z}_{n,2}, \ldots, \mathbf{z}_{n,N_t}]$ is the concatenation of all received vectors from all transmitters at the nth receiver, which is sent to the fusion center, where $\text{vec}(A)$ is obtained by stacking the columns of the matrix A on top of one another. At the fusion center, we denote $\mathbf{z} = \text{vec}([\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_{N_r}])$. Hence, we have

$$\mathbf{z} = \mathbf{H}\mathbf{s} + \mathbf{e}' \quad (8)$$

where $\mathbf{e}'$ denotes the equivalent additive noise vector including the noise $e'_{n,m}(t_{p,q})$, and $\mathbf{H}$ is defined by

$$\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_L] \quad (9)$$

$$\mathbf{h}_l = \text{vec}([\mathbf{h}_{l,1}, \mathbf{h}_{l,2}, \ldots, \mathbf{h}_{l,N_r}]) \quad (10)$$

$$\mathbf{h}_{l,n} = [\mathbf{h}_{l,n,1}, \mathbf{h}_{l,n,2}, \ldots, \mathbf{h}_{l,n,N_t}] \quad (11)$$

$$\mathbf{h}_{l,n,m} = \text{vec}([\mathbf{h}_{l,n,m,1}, \mathbf{h}_{l,n,m,2}, \ldots, \mathbf{h}_{l,n,m,P}]) \quad (12)$$

$$\mathbf{h}_{l,n,m,p} = [h_{\overline{\eta}_l,p,0,m,n}, \ldots, h_{\overline{\eta}_l,p,Q-1,m,n}]^T. \quad (13)$$

B. Off-Grid Approximations

Instead of the approximation in (5), some references, such as [32]–[36], proposed to use the first-order Taylor expansion of (4) around one of the neighbor grid-point

$$h_{\eta_k,p,q,m,n} \simeq h_{\overline{\eta}_{\ell(k)},p,q,m,n} + \frac{\partial h_{\overline{\eta}_{\ell(k)},p,q,m,n}}{\partial \overline{\eta}_{\ell(k)}} \delta_{\ell(k)} \quad (14)$$

where $\delta_{\ell(k)} = \eta_k - \overline{\eta}_{\ell(k)}$, $\overline{\eta}_{\ell(k)} \in \mathcal{S}$ is a neighbor grid-point and $\delta_{\ell(k)}$ is the deviation of the parameter from the grid point.

In this letter, instead of the approximations in (5) and (14), we propose to use the following linear convex approximation of (4) around both neighbor grid-points

$$h_{\eta_k,p,q,m,n} \simeq (1-\alpha_{\ell(k)}) h_{\overline{\eta}_{\ell(k)},p,q,m,n} + \alpha_{\ell(k)} h_{\overline{\eta}_{\ell(k)+1},p,q,m,n} \quad (15)$$

where $\alpha_{\ell(k)} = \frac{\delta_{\ell(k)}}{\Delta_{\ell(k)}} \in [0,1]$ is the normalized deviation from the neighboring grid-point, and $\Delta_{\ell(k)} = \overline{\eta}_{\ell(k)+1} - \overline{\eta}_{\ell(k)}$. Using (15) and same steps as in the previous sections, we obtain

$$\mathbf{z} = \mathbf{H}\mathbf{s} + \mathbf{H}_d(\mathbf{s} \odot \boldsymbol{\alpha}_n) + \mathbf{e}'' \quad (16)$$

where $\boldsymbol{\alpha}_n = [\alpha_1, \ldots, \alpha_{L-1}]^T$, and matrix $\mathbf{H}_d$ is defined as

$$\mathbf{H}_d = [\mathbf{h}_1^d, \ldots, \mathbf{h}_{L-1}^d] \quad (17)$$

$$\mathbf{h}_l^d = \mathrm{vec}([\mathbf{h}_{l,1}^d, \mathbf{h}_{l,2}^d, \ldots, \mathbf{h}_{l,N_r}^d]) \quad (18)$$

$$\mathbf{h}_{l,n}^d = \mathbf{h}_{l+1,n} - \mathbf{h}_{l,n}. \quad (19)$$

If $e''_{n,m}(t_{p,q})$ is defined to include the additive noise $e_{n,m}(t_{p,q})$ and the approximation errors in (15), the noise matrix $\mathbf{e}''$ is denoted by

$$\mathbf{e}'' = \mathrm{vec}([\mathbf{e}''_1, \mathbf{e}''_2, \ldots, \mathbf{e}''_{N_r}] \quad (20)$$

$$\mathbf{e}''_n = [\mathbf{e}''_{n,1}, \mathbf{e}''_{n,2}, \ldots, \mathbf{e}''_{n,N_t}] \quad (21)$$

$$\mathbf{e}''_{n,m} = \mathrm{vec}([\mathbf{e}''_{n,m,1}, \mathbf{e}''_{n,m,2}, \ldots, \mathbf{e}''_{n,m,P}]) \quad (22)$$

$$\mathbf{e}''_{n,m,p} = [e''_{n,m}(t_{p,0}), e''_{n,m}(t_{p,1}), \ldots, e''_{n,m}(t_{p,Q-1})]^T. \quad (23)$$

We aim to find the sparse vector $\mathbf{s}$ in (16). Interestingly, the element of vector $\mathbf{s} \odot \boldsymbol{\alpha}_n$ is zero if the corresponding element of $\mathbf{s}$ is zero. To exploit this block sparse property, we rewrite (16) as follows:

$$\mathbf{z} = \mathbf{H}''\mathbf{s}'' + \mathbf{e}'' \quad (24)$$

where $\mathbf{H}'' = [\mathbf{h}_1, \mathbf{h}_1^d, \ldots, \mathbf{h}_{L-1}, \mathbf{h}_{L-1}^d]$, and $\mathbf{s}''$ is defined by

$$\mathbf{s}'' = \mathrm{vec}\left([\mathbf{s}''[1], \ldots, \mathbf{s}''[L-1]]\right) \quad (25)$$

where $\mathbf{s}''[l] = [s_l, s_l\alpha_l]^T$. The number of nonzero vectors in $\{\mathbf{s}''[l]\}_{l=1}^{L-1}$ represents the number of targets.

III. CS for MIMO Radars

Since the vector $\mathbf{s}''$ in (24) is a $2K$-sparse vector, we may employ CS or random sampling in a MIMO radar by sending the reduced data $\mathbf{y}_n = \mathrm{vec}\left(\boldsymbol{\Phi}_n[\mathbf{z}_{n,1}, \mathbf{z}_{n,2}, \ldots, \mathbf{z}_{n,N_t}]\right)$ instead of $\mathbf{z}_n$ from the nth receiver to the fusion center [10], [40], where $\boldsymbol{\Phi}_n \in C^{M \times PQ}$ is a measurement matrix with $M < PQ$. Thus, for the received data at the fusion center, $\mathbf{y} = \mathrm{vec}\left([\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_{N_r}]\right)$, we can write

$$\mathbf{y} = \mathbf{A}\mathbf{s}'' + \mathbf{g} \quad (26)$$

where $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_1^d, \ldots, \mathbf{a}_{L-1}, \mathbf{a}_{L-1}^d]$ and

$$\mathbf{a}_l = \mathrm{vec}\left(\boldsymbol{\Phi}_l[\mathbf{h}_{l,1}, \mathbf{h}_{l,2}, \ldots, \mathbf{h}_{l,N_r}]\right) \quad (27)$$

$$\mathbf{a}_l^d = \mathrm{vec}\left(\boldsymbol{\Phi}_l[\mathbf{h}_{l,1}^d, \mathbf{h}_{l,2}^d, \ldots, \mathbf{h}_{l,N_r}^d]\right) \quad (28)$$

and $\mathbf{g} = \mathrm{vec}([\boldsymbol{\Phi}_1\mathbf{e}''_1, \boldsymbol{\Phi}_2\mathbf{e}''_2, \ldots, \boldsymbol{\Phi}_{N_r}\mathbf{e}''_{N_r}])$ is the additive noise using CS at the fusion center. The case where CS is not used, all $\boldsymbol{\Phi}_l$s are identity matrices.

IV. Block Sparse Recovery for Accurate Localization

By estimating the vectors $\{\mathbf{s}''[l]\}_{l=1}^{L-1}$, and then using $\mathbf{s}''[l] = s_l[1, \alpha_l]^T$, we can estimate the deviations of the detected targets from the grid points. A vector, such as $\mathbf{s}'' \in C^{dK}$, is defined in literature (e.g., see [41] and [42]) as a K-block sparse vector with block length of d when it is divided into L blocks with the length of d, only K blocks out of its L blocks are nonzero where usually $K \ll L$. As shown in (26), $\mathbf{s}''$ is a K-block sparse vector with the block length of $d = 2$ if there are K targets. Several algorithms have been proposed to exploit the block sparse property, which outperform the traditional sparse methods [6], [41]–[44]. Hence, for estimating $\mathbf{s}''$ from the measurement vector $\mathbf{y}$ in (26), any of the existing block sparse recovery methods can be used at the fusion center. In this letter for the simplicity, we do not consider strong clutter or jammer and use group Least Absolute Shrinkage and Selection Operator (LASSO) [45], [46] to estimate $\mathbf{s}''$. Group LASSO solves the following convex optimization problem:

$$\min_{\mathbf{s}''} \lambda \sum_{l=1}^{L-1} \| \mathbf{s}''[l] \|_2 + \|\mathbf{y} - \mathbf{A}\mathbf{s}''\|_2^2 \quad (29)$$

where λ is the relaxation parameter and should be carefully chosen. Note that the group LASSO algorithm with the block length of 1 is the traditional LASSO algorithm [8].

Assuming that the grid points are close enough such that no two targets are in the close vicinity of one of the grid points, we can neglect the neighbor points of a block with higher norm in the recovered signal. After finding the support of $\mathbf{s}''$, we can also estimate its entries by solving a reduced-size least-squares problem. Then, we propose to apply the following least-squares optimization based on (25) to estimate s_l and α_l for $l = 1, \ldots, L-1$:

$$\min_{\alpha_l \in [0,1], s_l \in C} |s''_{2l-1} - s_l|^2 + |s''_{2l} - s_l\alpha_l|^2. \quad (30)$$

the solution of the above problem must satisfy

$$s_l \leftarrow \frac{s''_{2l-1} + \alpha_l s''_{2l}}{1 + \alpha_l^2} \quad (31)$$

$$\alpha_l \leftarrow \begin{cases} \frac{\Re(s_l^* s''_{2l-1})}{|s_l|^2}, & \text{if } \frac{\Re(s_l^* s''_{2l-1})}{|s_l|^2} \in [0,1], \\ 1, & \text{if } \frac{\Re(s_l^* s''_{2l-1})}{|s_l|^2} > 1, \\ 0, & \text{if } \frac{\Re(s_l^* s''_{2l-1})}{|s_l|^2} < 0 \end{cases} \quad (32)$$

where $\Re(.)$ is the real part and $(.)^*$ is the conjugate of a complex number. The aforementioned equations have no close-form solution. Thus, we propose to use a simple iterative algorithm to find the solution by initializing $\alpha_l = \frac{1}{2}$ and updating s_l and δ_l via (31) and (32) iteratively. The convergence is easily proven since in each of the two updating steps of one iteration, the function (30) is minimized.

V. Simulation Results

In this section, we evaluate the performance of our proposed approach for targets localization. We consider a MIMO radar with the parameters mentioned in Table I.

The estimation space is uniformly discretized into either $L = 31$ grid points or $L = 51$ grid points, $\overline{\theta}_l$ for $l = 1, \ldots, L$, where $\overline{\theta}_l \in [0°, 20°]$, and $r_k \in [1000, 1200]$ m for $l = 1, \ldots, K$. We neglect the correlation between the transmit waveforms and

TABLE I
MIMO RADAR PARAMETERS FOR THE SIMULATIONS

Parameter	Value
The number of transmitters	$N_t = 10$
The number of receivers	$N_r = 10$
The locations of the transmitters	$d_m = 0.5(m-1)$ for $m = 1, \ldots, N_t$
The locations of the receivers	$\overline{d}_n = 0.05\,n$ for $n = 1, \ldots, N_r$
PRI	$T = 1$ ms
Pulses length	$T_p = 200\ \mu$s
Sampling period	$T_s = 1\ \mu$s
Pulse number	$P = 4$
The carrier frequency of the mth transmitter	$f_m = (50 + \frac{4m}{T_p})$ GHz
The transmit pulse of the mth transmitter	$s_m(t) = \mathrm{rect}\left(\frac{t-\frac{T}{2}}{T}\right) e^{j2\pi f_m}$
The number of samples	$Q = 100$
The initial point for sampling	$T_0 = \frac{2R_0}{c}$ where $R_0 = 1000$ m

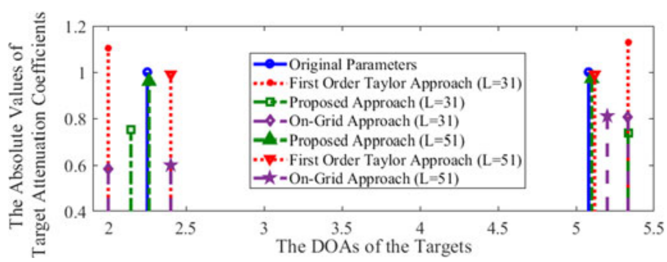

Fig. 1. Estimated DOAs versus the estimated absolute values of target attenuation coefficients where the NSNR is 0 dB and the sampling rate is 30%.

consider them to be completely separated at the receivers. We generate the noise entries by filtering a zero mean independent identically distributed Gaussian random sequence by the separating filters. We define sampling rate as $100\frac{M}{Q}(\%)$ and normalized signal-to-noise ratio (NSNR) as $10\log(\frac{\mathrm{E}\{\|\mathbf{y}\|^2\}}{K\,\mathrm{E}\{\|\mathbf{n}\|^2\}})$. For sparse recovery, we have used group LASSO in our approach and the first-order Taylor approach. In the on-grid approach, LASSO is used. Furthermore, we have used five iterations to find the solution of (29).

Fig. 1 shows the estimated DOAs versus the estimated absolute values of target attenuation coefficients by the on-grid approach, the first-order Taylor approach, and our proposed approach using random sampling at a 30% sampling rate and an NSNR of 0 dB. We see that the accuracy is increased by increasing the number of grid points, and our proposed approach results in a higher accuracy in the estimation of the target DOAs for both values of L.

In the next Monte Carlo experiment, we generate one target randomly in the interval $[0°, 20°]$ for 100 runs. Fig. 2 shows a comparison of the mean square error (MSE) of the estimated target DOAs using the on-grid approach, the first-order Taylor approach, and our proposed approach versus NSNR performed at a sampling rate of 30%. This figure also reveals that for enhancing the accuracy in target localization, we should increase the number of the grid points and the off-grid approaches have better performances for each value of L. Furthermore, the lowest MSE of DOA plot belongs to our proposed method.

Now, we compare the proposed method with the on-grid approach, the postprocessing remedies, and the first-order Taylor approach in terms of their detection probability. We simulate

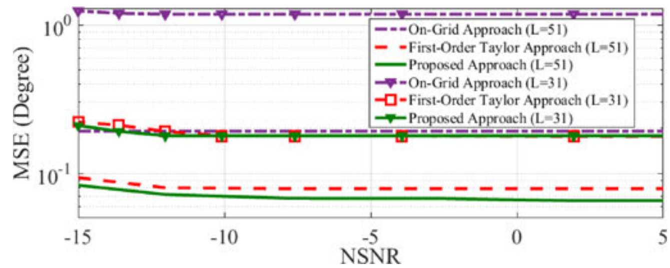

Fig. 2. MSE of the estimated DOAs versus NSNR for the sampling rate of 30%.

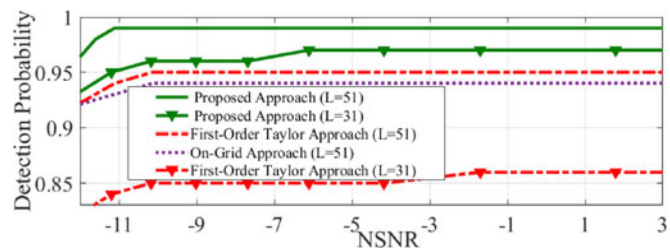

Fig. 3. Detection probability versus NSNR where the sampling rate is 30%.

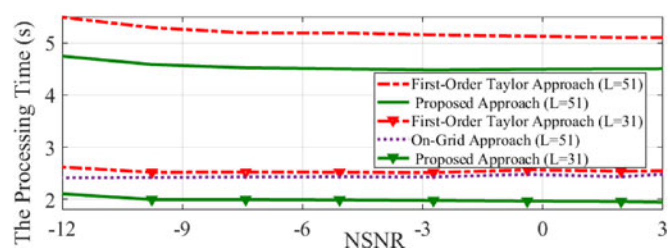

Fig. 4. Processing time versus NSNR for the sampling rate of 30%.

a tracking scenario for a single target randomly located on the search area. Fig. 3 shows the detection probability versus the NSNR for the on-grid approach using $L = 51$ and the first-order Taylor approach and the proposed approach using both mentioned grids ($L = 31$ and $L = 51$). In this figure, the sampling rate is 30% and detection is defined as the estimation of the target DOA with an absolute value of an error less than $0.4°$. As you can see, the proposed approach has a higher detection probability than its counterparts at the same NSNR. Furthermore, this approach can still be useful when the length of the estimation grid is reduced and even performs better than the on-grid approach using more grid points. Finally, the processing time of the mentioned approaches in Fig. 3 versus the NSNR is presented in Fig. 4, where the sampling rate is 30%. As shown in Figs. 3 and 4, the proposed approach can have a better performance in detection even when its complexity is less than its counterparts.

VI. CONCLUSION

In this letter, we have proposed a new off-grid approach for accurate localization in a MIMO radar using target sparsity. We have approximated the localization problem in a MIMO radar as a simple block sparse recovery problem that can be solved by any proper block sparse recovery method. By using the proposed approach, target DOAs can be precisely estimated and the detection probability can be increased compared to the counterpart approaches even when the complexity is reduced.

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