Opposing trends of geometrical parameters in maximisation of micro-ring resonator quality factor

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Opposing trends of geometrical parameters in minimisation of bending loss and thus in maximising the quality factor are briefly discussed for micro-ring resonators. It is shown that, while the quality factor of low order modes is an oscillatory function of geometrical parameters, the quality factor of high order modes is a monotonic function. The former has discrete pairs of optimum inner and outer ring radii which maximises the quality factor. In contrast, the quality factor of the latter has no local maximum. Introduction of slight inhomogeneities does not change the overall behaviour of the quality factor but can increase its overall level when the refractive index of the ring region increases at its outer radius.

Introduction: Micro-ring resonators have been widely used in miscellaneous applications including optical filtering, multiplexing, signal processing, bio-sensing, and optical switching. Since high quality factors are desirable in most of these applications, several studies have already been carried out on the influence of the refractive index profile, polarisation, average radius and the thickness of the ring on the quality factor [1–4]. Nevertheless, the optimum parameters ensuring the minimum level of loss in micro-ring resonators have yet to be found.

There are three different loss mechanisms affecting the overall quality factor of a micro-ring resonator: material intrinsic loss (absorption loss), surface scattering, and bending loss. Material absorption compared against the others is usually negligible. It is, for instance, much smaller than 0.1 dB/cm for a micro-ring resonator made of pure silicon at the wavelength of $\lambda = 1.55$ μm [5]. Surface scattering caused by the likely to occur imperfections on the ring surface, e.g. by dust or scratches, is reduced by high quality fabrication methods and thus can be also neglected. Bending loss mechanism is on the other hand due to the curvature of the micro-ring resonator and is therefore inevitable. This is even more of a fact for small ring resonators, the bending loss mechanism of which dominates the other two. This Letter is for this reason devoted to the maximisation of the quality factor by assuming that the bending loss is dominant. Because of the complex nature of optimising a three-dimensional micro-ring resonator, however, the optimum geometrical parameters to ensure minimum bending loss are here found for two-dimensional micro-ring resonators. To this end, the dependence of the bending loss and thus the quality factor upon the geometrical parameters (micro-ring radius and its thickness) of two-dimensional homogeneous/inhomogeneous micro-ring resonators is here studied by solving the Helmholtz equation.

Results and discussion: The geometrical structure of the here-studied micro-ring resonator is schematically shown in Fig. 1*a*. Two refractive index profiles, a piecewise homogenous and a linearly inhomogeneous shown in Fig. 1*b* are then considered. The desired resonance wavelength is then set at $\lambda = 1.55$ μm.

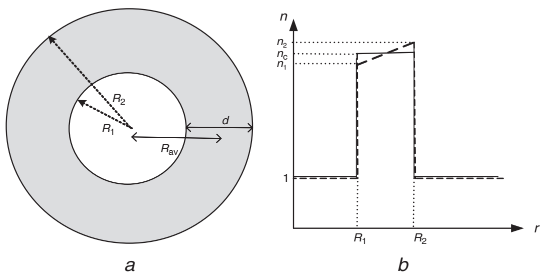

Fig. 1 *Schematic of micro-ring resonator, and refractive index profiles of homogenous and linearly homogeneous ring resonator*

a Schematic of micro-ring resonator, average radius, R_{av}, is $R_{av} = (R_1 + R_2)/2$; thickness of ring, d is $d = R_1 - R_2$
b Refractive index profiles homogenous (solid line) and linearly inhomogeneous (dashed line) ring resonator

First, the more typical case of the homogeneous micro-ring resonator is studied. In accordance with Fig. 1, the ring refractive index is set to $n_c = 3.5$ and then a typical low order E-polarised mode, $TE_{0,6}$, and a typical high order E-polarised mode, $TE_{11,4}$, are chosen to have the desired resonance wavelength. The average radius of the micro-ring resonator, R_{av}, and the thickness of the ring, d, for the $TE_{0,6}$ and the $TE_{11,4}$ modes supporting the desired resonance wavelength are then plotted against each other in Figs 2*a* and *b*, respectively. These Figures show that the appropriate thickness of the ring is for both low and high order modes decreased by increasing the average radius of the ring. This is not surprising because the effect of increasing the average radius of the ring in increasing the density of resonant modes should be countered by decreasing the thickness of the ring in order to keep the resonance wavelength fixed at $\lambda = 1.55$ μm. The quality factor of the pairs of R_{av} and d supporting the desired resonance wavelength is also plotted in Figs 2*a* and *b*. Thereby, the contrast between the low and higher order modes is revealed. Fig. 2*a* shows that, even though there are an uncountable number of (R_{av},d) pairs to set the resonance wavelength of the $TE_{0,6}$ mode fixed at $\lambda = 1.55$ μm, there are only countable pairs of (R_{av},d) whose quality factors are locally maximum. There are only five pairs of such (R_{av},d) in our case. Interestingly, the optimum (R_{av},d) pairs with larger average radius, i.e. larger size, have larger quality factor. It is therefore recommended to look for the optimum pair of (R_{av},d) in a careful design of a micro-ring resonator with a low order resonance mode. Fig. 2*b*, on the other hand, shows that the quality factor of the $TE_{11,4}$ mode resonating at the fixed wavelength of $\lambda = 1.55$ μm has no local maxima. It is rather monotonically decreasing by increasing the average radius and by decreasing the thickness of the ring. It is therefore recommended to have the largest possible thickness of the ring and the lowest possible average radius to ensure a high quality factor. Fig. 2*b* also shows that higher order modes have considerably larger quality factors than lower order modes.

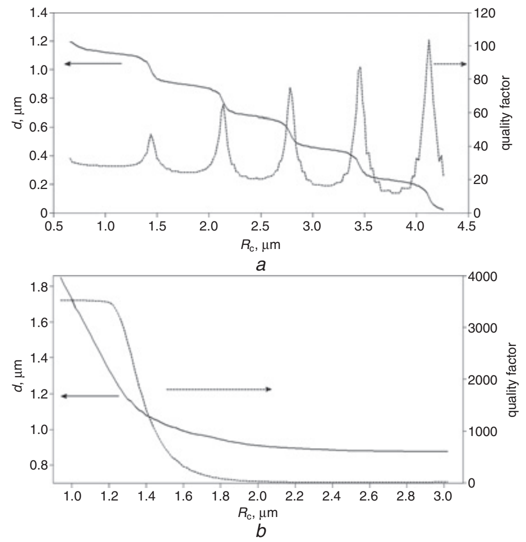

Fig. 2 *Thickness (solid line) and quality factor (dashed line) of typical homogeneous ring with $TE_{0,6}$, and $TE_{11,4}$ resonance wavelength λ = 1.55 μm against average radius of ring*

a $TE_{0,6}$
b $TE_{11,4}$

Secondly, the micro-ring resonator with a linearly inhomogeneous refractive index profile is considered. In accordance with Fig. 1*b*, we set $n_1 = 3.4$ and n_2 is a specified value such that the average refractive index of the ring region is:

$$n_{av} = \frac{\int_{R1}^{R2}\int_0^{2\pi} n(r) r dr d\varphi}{\int_{R1}^{R2}\int_0^{2\pi} r dr d\varphi} = n_c$$

The desired resonance wavelength is once again set at $\lambda = 1.55$ μm and then the average radius of the micro-ring resonator, R_{av}, and the thickness of the ring, d, for the $TE_{0,6}$ and the $TE_{11,4}$ modes are plotted against each other in Figs 3*a* and *b*, respectively. Similarly and for the very same reason, the appropriate thickness of the ring is for both low and high order modes decreased by increasing the average radius of the ring. The dependence of the quality factor of the $TE_{0,6}$ and the $TE_{11,4}$ modes on the ring radii in the inhomogeneous micro-ring resonator is quite similar to that of the $TE_{0,6}$ and the $TE_{11,4}$ modes in the former homogeneous micro-ring resonator. Nevertheless, the quality factor of the inhomogeneous resonator is slightly larger than that of the homogeneous one. Since the average ring refractive index of the inhomogeneous resonator, n_{av}, equals the ring refractive index of the homogeneous resonator, n_c, the higher quality factor of the inhomogeneous resonator can be attributed to the fact that the radiative field is warded off by increasing the refractive index of the ring region at the outer radius of the micro-ring resonator.

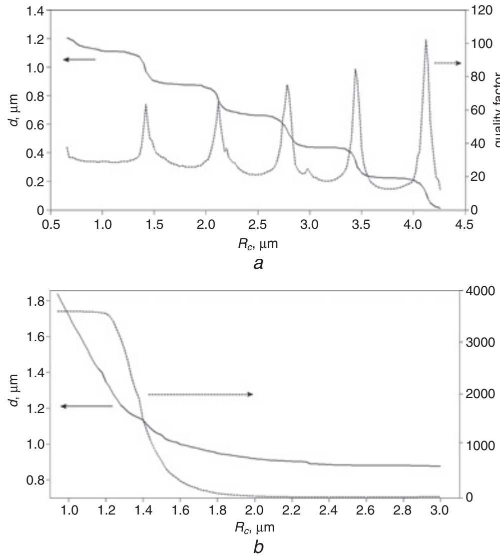

Fig. 3 *Thickness (solid line) and quality factor (dashed line) of typical inhomogeneous ring with $TE_{0,6}$, and $TE_{11,4}$ resonance wavelength $\lambda = 1.55$ µm against average radius of ring*

a $TE_{0,6}$
b $TE_{11,4}$

Conclusion: The influence of geometrical parameters in determining the quality factor of micro-ring resonators depends on the mode number and does not conspicuously change by the introduction of inhomogeneities in the refractive index profile of the ring. The quality factor of lower order modes is an oscillatory function of the geometrical parameters while the quality factor of higher order modes is a monotonic one. Increasing the ring refractive index at its outer interface is also helpful in increasing the resonator quality factor.

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