Performance analysis and comparison of different multirate TH-UWB systems: uncoded and coded schemes

M. Nasiri-Kenari and M.G. Shayesteh

Abstract: Multirate time hopping ultra wideband (TH-UWB) communication systems are considered where each user can have several multirate services. Three methods named as parallel mapping, serial mapping with variable spreading factor, and multicode using Walsh subcode concatenation schemes are investigated. In the parallel mapping method, one kind of signature code (PN code) is used to distinguish users and their diverse services alike. For the serial mapping with variable spreading factor scheme, one signature code is assigned to each user and different services send their data in distinct frames. Whereas in the multicode using Walsh subcode method, different users and different services of the same user are distinguished through two kinds of signature codes, namely PN codes and Walsh codes, respectively. The performance of these three multirate TH-UWB systems is evaluated and compared for both uncoded and coded schemes in synchronous AWGN channel using conventional single user correlator receiver. The analytical and numerical results indicate that while the multicode using Walsh subcode structure outperforms the other structures in the uncoded system, the parallel mapping method performs the best for the coded scheme. The results also show that even though the coded scheme considered requires the same bandwidth as the uncoded scheme, the coded scheme significantly outperforms the uncoded scheme for all three multirate structures.

1 Introduction

TH-UWB systems are spread spectrum systems first introduced in [1, 2]. In this system, data is transmitted using a time-hopping signal composed of sub-nanosecond pulses. Because of using sub-nanosecond pulses, the power spectrum of the system has an ultra-wide bandwidth about several gigahertz and very low density well below the thermal noise floor. As a result, the TH-UWB systems do not interfere much with other narrow bandwidth systems operating in the same band. The capability of the system to highly resolve the multipaths with differential delays on the order of one nanosecond or less and its ability to penetrate materials, make the system viable for high–quality, fully mobile short-range indoor radio communication systems [1–7]. Because of its above advantages, and also its low power assumption, low probability of detection, low cost, and, finally, nearly 'all digital' transmitter/receiver, the TH-UWB communication system has received great amount of attentions in recent years. For some recent works on TH-UWB systems, see [8–12].

In [3], a practical low-rate coding scheme is applied to the TH-UWB system, which does not require any extra bandwidth other than what is needed by TH-spread spectrum modulation. The error correcting code used for demonstrating the coded system performance was a super-orthogonal code [13] with rate $1/N_s$, where N_s is the number of pulses transmitted for each bit in the TH-UWB system. The system performance analysis in [3, 4] indicates that the coded scheme outperforms the uncoded scheme significantly or more importantly, at a given bit error rate, the coding scheme increases the number of users by a factor, which is logarithmic in N_s, as defined above.

In this paper, we consider multirate-multiservice UWB systems where users have several services with the same or different rates. The idea of multirate spread spectrum systems has been mostly studied for DS-CDMA or MC-CDMA systems, for example see [14–20]. However, to the best of our knowledge, the current work is the first to consider multirate-multiservice TH-UWB systems.

We investigate three methods for this purpose for the uncoded and coded schemes. In the first method, parallel mapping, each service of user is considered as a virtual user and a distinct pseudorandom (PN) code is assigned to the service. So the different users and the services of the same user are distinguished by distinct PN codes. In the second method, serial mapping with variable spreading factor, the users are differentiated by distinct PN codes. Then, the services of the same users have the same PN code but the services send their data in distinct frames. The number of frames assigned to each service will be variable depending on the rate. If the services have the same rates, then the number of frames assigned to each service is equal. In the third method, which we name multicode using Walsh subcode concatenation scheme, different users and different services of the same user are distinguished through two kinds of signature codes, namely PN codes and Walsh

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M. Nasiri-Kenari is with Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran

M.G. Shayesteh is with Department of Electrical Engineering, Urmia University, Urmia, Iran, and the Wireless Research Laboratory, Electrical Engineering Department, Sharif University of Technology, Tehran, Iran

E-mail: m.shayesteh@mail.urmia.ac.ir

codes, respectively. That is, the services of the same user are assigned the same PN code, but the services are differentiated by orthogonal Walsh codes. Throughout this paper, we assume that each service of each user has the same rate $1/T$. The results can easily be generalised to the multirate case. Note that the multirate services with bit rates higher than $1/T$ are easily supplied by either using more than one code for each service in the parallel mapping and multicode methods or assigning different numbers of frames to each service in the serial mapping structure.

We evaluate the bit error rate of the three methods in uncoded and coded schemes over a synchronous AWGN channel using the conventional correlator receiver. The results show that for the uncoded scheme, the multicode using Walsh subcode concatenation scheme outperforms the other two methods while the serial mapping method has better performance than the parallel mapping one. We consider the coded scheme of the three methods using super-orthogonal codes. We compute the lower and upper bounds on the bit error rate for the coded schemes. Then, a comparison between the uncoded and coded schemes is presented for all the three structures. It is shown that the coded scheme significantly outperforms the uncoded scheme. The results also show that for the coded scheme, the parallel mapping method has the best performance. The performance analysis in fading channel for the above three multirate techniques is under investigation.

2 System models

In this Section, we explain the model of the UWB system and the different multirate, multiservice structures for this system.

2.1 UWB system description

In the UWB system considered, as in [1–7], the duration of each bit is divided into N_s frames each with duration T_f. In each frame one pulse with duration less than 1 ns is transmitted, so the bit rate is equal to $R_s = 1/T = 1/N_sT_f$. The modulation is binary pulse position modulation (BPPM), in which the pulses corresponding to bit 1 are sent δ seconds later than the pulses corresponding to bit -1. Location of the pulses in each frame is determined by the user dedicated pseudorandom sequence. The received waveform of user k is

$$S^k_{rec}(t) = \sum_j w_{rec}\left[t - jT_f - c^k_j T_c - \left(\frac{d^k_j + 1}{2}\right)\delta\right] \quad (1)$$

where the index j indicates the frame number, $w_{rec}(t)$ represents the received monocycle pulse with the duration of T_w which satisfies the relation $\int_{-\infty}^{\infty} w_{rec}(t)dt = 0$, and $\{c^k_j\}$ is the dedicated pseudorandom sequence for user k with integer components, which can take on values between 0 to N_h−1 uniformly, T_c is the chip duration and satisfies $N_hT_c \leq T_f$, and $\{d^k_j\} \in \pm 1$ is the binary sequence of the transmitted symbols corresponding to user k. For the uncoded system, this sequence is N_s repetitions of the transmitted data sequence, i.e. if the transmitted binary data sequence is $\{D^k_i\}$, then we have

$$d^k_j = D^k_i \quad \text{for} \quad i = \left[\frac{j}{N_s}\right] \quad (2)$$

where $[x]$ denotes the largest integer not greater than x.

2.2 Parallel mapping method

In this method, each service of user is considered as a virtual user and is assigned a distinct pseudorandom sequence. Therefore, the users and the services of the same users are distinguished by the assigned different PN codes. The received signal in synchronous AWGN channel can be written as

$$r(t) = \sum_{k=1}^{K}\sum_{q=1}^{Q} s^{k,q}_{rec}(t) + n(t) = \sum_{k=1}^{K}\sum_{q=1}^{Q}\sum_j A^{k,q} w_{rec}\left[t - jT_f - c^{k,q}_j T_c - \left(\frac{d^{k,q}_j + 1}{2}\right)\delta\right] + n(t) \quad (3)$$

where K and Q are the numbers of independent users and services of each user, respectively, $A^{k,q}$ is the amplitude of the qth service of kth user's signal, $d^{k,q}_j$, defined as in (2), is the binary sequence of the transmitted symbols of the qth service of user k, and $n(t)$ is the additive white Gaussian noise (AWGN) with two sided power spectral density equal to $N_0/2$. In this method, we can have interference from all services of other users and also the other services of the same user to the desired service.

2.3 Serial mapping using variable spreading factor structure

In this structure, one signature code is assigned to each user and different services send their data in distinct frames. That is, the duration of each bit, which consists of N_s frames, is divided into Q, and each service of user transmits its data in N_s/Q frames separately (we assume the services have the same rate). If we consider different rates for different services, then the number of frames assigned to each service will be variable depending on the rate. However, there is no interference from the other services of the same user to the desired service. For the case that the services have the same rate, only one service of each interfering user interferes with the desired service. The received signal can be written as

$$r(t) = \sum_{k=1}^{K}\sum_j B^{k,\mathrm{mod}(j,Q)} w_{rec}\left[t - jT_f - c^k_j T_c - \left(\frac{d^{k,\mathrm{mod}(j,Q)}_j + 1}{2}\right)\delta\right] + n(t) \quad (4)$$

where $\mathrm{mod}\{a, b\}$ denotes the reminder of division a to b, and $B^{k,\mathrm{mod}(j,Q)}$ is the amplitude of the $\mathrm{mod}(j, Q)$ service of user k. Note that for uncoded scheme: $d^{k,\mathrm{mod}(j,Q)}_j = D^{k,\mathrm{mod}(j,Q)}_i$ where $i = [j/N_sQ]$.

2.4 Multicode using Walsh subcode concatenation scheme

In this method, we use two different codes for distinguishing users and their services. The first code is a PN sequence which is used to separate different users and the second one is the Walsh code which is applied to distinguish the services of each user. That is, in this method, all the services of a user utilise the same chips, determined by the user PN code, for sending their N_s pulses. To separate different services of the user, a second orthogonal Walsh code with length N_s is dedicated to each service. The received signal of the qth service of user k can be written as

$$s^{k,q}_{rec}(t) = \sum_j A^{k,q} w_{rec}\left[t - jT_f - c^k_j T_c - \left(\frac{d^{k,q}_j h^{k,q}_j + 1}{2}\right)\delta\right] \quad (5)$$

where $h^{k,q}_j \in \{\pm 1\}$ is the Walsh code with period of N_s, i.e.

$h_{j+N_s}^{k,q} = h_j^{k,q}$ and,

$$\langle \bar{h}^{k,q} \bar{h}^{k,q'} \rangle \triangleq \sum_{j=0}^{N_s-1} h_j^{k,q} h_j^{k,q'} = \begin{cases} 0 & q \neq q' \\ N_s & q = q' \end{cases} \tag{6}$$

It is easily verified from (6) that the number of $+1$s and -1s in the multiplication of two different Walsh codes is $N_s/2$. The maximum number of services with different Walsh codes is N_s so we should have $Q < N_s$. We also observe from (5) that both the Walsh code and data determine the shift of δ seconds of pulse in each frame. The received signal can be written as

$$r(t) = \sum_{k=1}^{K} \sum_{q=1}^{Q} A^{k,q} \sum_{j} w_{rec}\left[t - jT_f - c_j^k T_c - \left(\frac{d_j^{k,q} h_j^{k,q} + 1}{2}\right)\delta\right] + n(t) \tag{7}$$

Note that even though different services of the same user have different Walsh subcodes, it is possible that services of different users have the same Walsh code, which happens when the number of Walsh codes (N_s) is less than the total number of services (KQ). We consider this possibility in our performance evaluation.

An example of the transmitted signal for each scheme is presented in Fig. 1.

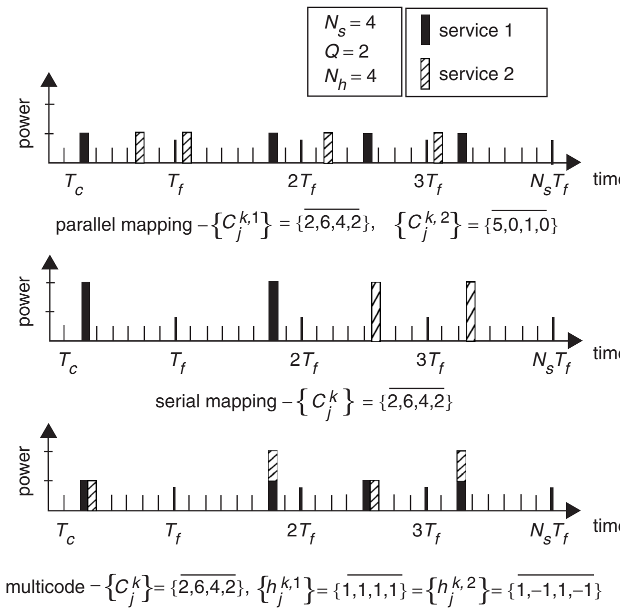

Fig. 1 *Transmitted signals in one bit duration of the three methods (uncoded schemes), for energy per service-bit of four and services information bits of* $D_i^{k,1} = -1$ *and* $D_i^{k,2} = 1$

3 Performance analysis of uncoded schemes

In this Section, we first evaluate the bit error rate of multicode using the Walsh subcode concatenation method. The bit error rate of parallel and serial mapping structures will be calculated from the multicode method by proper parameter replacing. Then, we compare their performance analytically.

3.1 Multicode using Walsh subcode concatenation method

Without loss of generality, we assume that the first service of the first user is the desired one. Then, the conventional correlator receiver makes decision based on the following rule

$$D_0^{1,1} = 1 \text{ if}$$
$$\int_0^T r(t) \sum_{j=1}^{N_s-1} w_{rec}\left[t - jT_f - c_j^1 T_c - \left(\frac{h_j^{1,1}+1}{2}\right)\delta\right] dt > \int_0^T r(t) \sum_{j=0}^{N_s-1} w_{rec}\left[t - jT_f - c_j^1 T_c - \left(\frac{-h_j^{1,1}+1}{2}\right)\delta\right] dt \tag{8}$$

The above relation can be written as

$$\sum_{j=0}^{N_s-1} \int_{jT_f}^{(j+1)T_f} r(t)\left\{w_{rec}\left[t - jT_f - c_j^1 T_c - \left(\frac{h_j^{1,1}+1}{2}\right)\delta\right] - w_{rec}\left[t - jT_f - c_j^1 T_c - \left(\frac{-h_j^{1,1}+1}{2}\right)\delta\right]\right\} dt > 0 \tag{9}$$

In order to calculate the bit error rate, we obtain the output of the receiver owing to the desired signal, interference and noise.

3.1.1 The output owing to the desired user:

The output owing to the first service of user one is obtained by replacing $s_{rec}^{1,1}(t)$ instead of $r(t)$ in (9) which results in

$$S = A^{1,1} \sum_{j=0}^{N_s-1} \int_{jT_f}^{(j+1)T_f} w_{rec}\left[t - jT_f - c_j^1 T_c - \left(\frac{d_j^{1,1} h_j^{1,1}+1}{2}\right)\delta\right] \cdot \left\{w_{rec}\left[t - jT_f - c_j^1 T_c - \left(\frac{h_j^{1,1}+1}{2}\right)\delta\right] - w_{rec}\left[t - jT_f - c_j^1 T_c - \left(\frac{-h_j^{1,1}+1}{2}\right)\delta\right]\right\} dt \tag{10}$$

By a change of variable $t - jT_f \rightarrow t$, we obtain

$$S = A^{1,1} \sum_{j=0}^{N_s-1} \int_0^{T_f} w_{rec}\left[t - c_j^1 T_c - \left(\frac{d_j^{1,1} h_j^{1,1}+1}{2}\right)\delta\right] \times \left\{w_{rec}\left[t - c_j^1 T_c - \left(\frac{h_j^{1,1}+1}{2}\right)\delta\right] - w_{rec}\left[t - c_j^1 T_c - \left(\frac{-h_j^{1,1}+1}{2}\right)\delta\right]\right\} dt \tag{11}$$

Since $w(t)$ is nonzero in the interval $[0, T_w]$, we have

$$w_{rec}\left[t - c_j^1 T_c - \left(\frac{h_j^{1,1}+1}{2}\right)\delta\right] \in \left[c_j^1 T_c,\ c_j^1 T_c + T_w + \delta\right] \tag{12}$$

Again we use a change of variable $t - c_j^1 T_c \rightarrow t$, then (11) will be reduced to

$$S = A^{1,1} \sum_{j=0}^{N_s-1} \int_0^{T_W+\delta} w_{rec}\left[t - \left(\frac{d_j^{1,1} h_j^{1,1} + 1}{2}\right)\delta\right] \times \left\{ w_{rec}\left[t - \left(\frac{h_j^{1,1}+1}{2}\right)\delta\right] - w_{rec}\left[t - \left(\frac{-h_j^{1,1}+1}{2}\right)\delta\right]\right\} dt \quad (13)$$

Now, we obtain the output owing to the transmitted bit equal to $d_j^{1,1} = 1$. We compute the above integral considering different values of $h_j^{1,1}$.

a) $h_j^{1,1} = 1$, then the result of integral is

$$\int_0^{T_W+\delta} w_{rec}(t-\delta)[w_{rec}(t-\delta) - w_{rec}(t)]dt = \int_0^{T_W+\delta} w_{rec}(t)[w_{rec}(t) - w_{rec}(t-\delta)dt] \triangleq m_p \quad (14)$$

b) $h_j^{1,1} = -1$. It is easily verified that the result of integral is again equal to m_p, so the output is

$$S/(d_j^{1,1} = 1) = A^{1,1} N_s m_p \quad (15)$$

Similarly, we obtain the output for the transmitted data being equal to -1 as

$$S/(d_j^{1,1} = -1) = -A^{1,1} N_s m_p \quad (16)$$

3.1.2 The effect of self interference (SI): Here, we obtain the output of the receiver owing to the other services of the first user. Replacing $s_{rec}^{1,q}(t)$ instead of $r(t)$ in (9) gives

$$SI^q = A^{1,q} \sum_{j=0}^{N_s-1} \int_0^{T_w+\delta} w_{rec}\left[t - \left(\frac{d_j^{1,q} h_j^{1,q} + 1}{2}\right)\delta\right] \times \left\{ w_{rec}\left[t - \left(\frac{h_j^{1,1}+1}{2}\right)\delta\right] - w_{rec}\left[t - \left(\frac{-h_j^{1,1}+1}{2}\right)\delta\right]\right\} dt \quad (17)$$

For the transmitted data equal to $+1$ we have

$$SI^q/(d_j^{1,q} = 1) = A^{1,q} \sum_{j=0}^{N_s-1} \int_0^{T_w+\delta} w_{rec}\left[t - \left(\frac{h_j^{1,q} + 1}{2}\right)\delta\right] \times \left\{ w_{rec}\left[t - \left(\frac{h_j^{1,1}+1}{2}\right)\delta\right] - w_{rec}\left[t - \left(\frac{-h_j^{1,1}+1}{2}\right)\delta\right]\right\} dt \quad (18)$$

Since $\sum_{j=0}^{N_s-1} h_j^{k,q} h_j^{k,q'} = 0$ for $q \neq q'$, we conclude that the number of values q for which $h_j^{1,1}$ and $h_j^{1,q}$ have the same sign equals the number for which they have a different sign. For $h_j^{1,1}$ and $h_j^{1,q}$ with the same sign, we can obtain that the above integral is m_p and for different signs, the integral is $-m_p$. So, the self interference (SI) is

$$SI^q/(d_j^{1,q} = 1) = \frac{N_s}{2} A^{1,q}[m_p - m_p] = 0 \quad (19)$$

Similarly we can obtain

$$SI^q/(d_j^{1,q} = -1) = 0 \quad (20)$$

Therefore, even though the self interference in each frame is not zero and equals either $A^{1,q}m_p$ or $-A^{1,q}m_p$, the total interference in one bit duration (N_s frames) is zero because of the orthogonal property of Walsh codes.

3.1.3 Interference from the other users (MAI): The output of the receiver owing to the qth service of the kth user (multiple access interference) is

$$I^{k,q} = \sum_{j=0}^{N_s-1} I_j^{k,q} \quad (21)$$

where

$$I_j^{k,q} \equiv A^{k,q} \int_0^{T_f} w_{rec}\left[t_f - c_j^k T_c - \left(\frac{d_j^{k,q} h_j^{k,q} + 1}{2}\right)\delta\right] \times \left\{ w_{rec}\left[t - c_j^1 T_c - \left(\frac{h_j^{1,1}+1}{2}\right)\delta\right] - w_{rec}\left[t - c_j^1 T_c - \left(\frac{-h_j^{1,1}+1}{2}\right)\delta\right]\right\} dt \quad (22)$$

If $c_j^k \neq c_j^1$, then the above integral is zero because the first user's pulse and that of the kth user do not overlap, so $I_j^{k,q} = 0$. The probability that the two PN codes have the same or different values in the frame j is

$$p(c_j^k = c_j^1) = \beta = \frac{1}{N_h}; \quad p(c_j^k \neq c_j^1) = \alpha = 1 - \frac{1}{N_h} \quad (23)$$

Now, we evaluate (22) when the components of the PN codes of the two users in frame j are equal, i.e. $c_j^k = c_j^1$, as follows

$$I_j^{k,q} \equiv A^{k,q} \int_0^{T_w+\delta} w_{rec}\left[t - \left(\frac{d_j^{k,q} h_j^{k,q} + 1}{2}\right)\delta\right] \times \left\{ w_{rec}\left[t - \left(\frac{h_j^{1,1}+1}{2}\right)\delta\right] - w_{rec}\left[t - \left(\frac{-h_j^{1,1}+1}{2}\right)\delta\right]\right\} dt \quad \text{if } c_j^k = c_j^1 \quad (24)$$

As mentioned before, there exists the possibility that one service of some other users has the same Walsh code as the service 1 of user 1 (desired service) which happens when $N_s < KQ$. Note that in that case, the other services of the user will have definitely different Walsh codes with the desired service. Therefore, to compute (24), we consider the two cases separately.

a) Assume service q of interfering user k has the same Walsh code as the desired service, i.e. $h_j^{k,q} = h_j^{1,1}\ \forall j$. It can easily be verified from (13)–(16) that

$$I_j^{k,q}/(d_j^{k,q}=1)=A^{k,q}m_p;\quad I_j^{k,q}/(d_j^{k,q}=-1)=-A^{k,q}m_p \quad \text{if } c_j^k=c_j^1 \tag{25}$$

Therefore, from (23)–(25), the probability density functions of the interference owing to this service in one frame are easily obtained as

$$\begin{aligned} P_{I_j^{k,q}/(d_j^{k,q}=1)}(x) &= \alpha\delta(x)+\beta\delta(x-A^{k,q}m_p);\\ P_{I_j^{k,q}/(d_j^{k,q}=-1)}(x) &= \alpha\delta(x)+\beta\delta(x+A^{k,q}m_p) \end{aligned} \tag{26}$$

The characteristic function [21] is obtained by applying the z transform to the above equation

$$\begin{aligned} \varphi_{I_j^{k,q}/(d_j^{k,q}=1)}(z) &= \alpha+\beta z^{-A^{k,q}m_p};\\ \varphi_{I^{k,q}/(d_j^{k,q}=-1)}(z) &= \alpha+\beta z^{A^{k,q}m_p} \end{aligned} \tag{27}$$

Since c_j^ks are assumed to be pseudorandom, the receiver outputs in different frames are independent. So, the conditional characteristic functions of the total interference caused by the interfering service in one bit duration (N_s frames) given the data are

$$\begin{aligned} \varphi_{I^{k,q}/(d_j^{k,q}=1)}(z) &= \left(\alpha+\beta z^{-A^{k,q}m_p}\right)^{N_s};\\ \varphi_{I^{k,q}/(d_j^{k,q}=-1)}(z) &= \left(\alpha+\beta z^{A^{k,q}m_p}\right)^{N_s} \end{aligned} \tag{28}$$

Therefore, the unconditional characteristic function is obtained as

$$\begin{aligned} \varphi_{I^{k,q}}(z) =& p(d_j^{k,q}=1)\varphi_{I_j^{k,q}/(d_j^{k,q}=1)}(z)\\ &+p(d_j^{k,q}=-1)\varphi_{I_j^{k,q}/(d_j^{k,q}=-1)}\\ =&\frac{1}{2}\left(\alpha+\beta z^{-A^{k,q}m_p}\right)^{N_s}+\frac{1}{2}\left(\alpha+\beta z^{A^{k,q}m_p}\right)^{N_s} \end{aligned} \tag{29}$$

b) If we assume that U services (each belonging to the different users) have the same Walsh code as the desired service, then the $(K-1)Q-U$ services will have different Walsh codes with the desired service. In this case, the output owing to each of these services in one frame is the same as that of the self interference (computed in Section 3.1.2), so we have

$$\begin{aligned} I_j^{k,q}/(d_j^{k,q}=1) =& I_j^{k,q}/(d_j^{k,q}=-1)\\ =&\begin{cases} 0 & c_j^k\neq c_j^1\\ \pm A^{k,q}m_p & c_j^k=c_j^1 \end{cases} \end{aligned} \tag{30}$$

where ± 1 depends on the sign of the product $h_j^{1,1}\cdot h_j^{k,q}$. So the probability density function in each frame is

$$\begin{aligned} P_{I_j^{k,q}}(x) &= \alpha\delta(x)+\beta\delta(x+A^{k,q}m_p) \quad \text{or}\\ P_{I_j^{k,q}}(x) &= \alpha\delta(x)+\beta\delta(x-A^{k,q}m_p) \end{aligned} \tag{31}$$

Considering the paragraph right after (18), the characteristic function of the total interference owing to all N_s frames is obtained as

$$\varphi_{I^{k,q}}(z)=\left(\alpha+\beta z^{-A^{k,q}m_p}\right)^{N_s/2}\left(\alpha+\beta z^{A^{k,q}m_p}\right)^{N_s/2} \tag{32}$$

Let E be the set of services with the same Walsh code as the desired service with the cardinality of U and E' be the complement of this set with the cardinality of $(K-1)Q-U$. Then, the total interference owing to all interfering services is

$$\begin{aligned} MAI &= \sum_{k=2}^{K}\sum_{q=1}^{Q}\sum_{j=0}^{N_s-1} I_j^{k,q}\\ &= \sum_{e=1}^{(K-1)Q} MAI(e) = MAI_E + MAI_{E'} \end{aligned} \tag{33}$$

where MAI_E and $MAI_{E'}$ are the interferences caused by the services in sets E and E', respectively. Now, we obtain the mean and variance of MAI. From the above equation, we have

$$E(MAI)=\sum_{k=1}^{K}\sum_{q=2}^{Q}\sum_{j=0}^{N_s-1}E(I_j^{k,q}) \tag{34}$$

where $E(.)$ represents expectation. From (26) and (30), it is observed that $I_j^{k,q}$ (the interference in each frame) can take three possible values, so we easily obtain:

$$\begin{aligned} E(I_j^{k,q}) &= 0\times\alpha+\frac{\beta}{2}\times(A_j^{k,q}m_p-A_j^{k,q}m_p)\Rightarrow E(MAI)=0\\ &=0 \end{aligned} \tag{35}$$

As mentioned before, when one service of some users has the same Walsh code as the desired service, then the other services of those users have different Walsh codes. Therefore, there is not independence between the two groups of interference because of the services belonging to the different sets E and E'. However, the input data of different services, i.e., $d_j^{k,q}$s, are independent from each other, and they are also independent from the PN and Walsh codes. So, from (33), we can conclude that the interferences are uncorrelated. Then, we have

$$\begin{aligned} \sigma^2_{MAI|U} &= \sigma^2_{MAI_E|U}+\sigma^2_{MAI_{E'}|U};\\ \sigma^2_{MAI_E|U} &= \sum_{e\in E}\sigma^2_{MAI}(e);\\ \sigma^2_{MAI_{E'}|U} &= \sum_{e'\in E'}\sigma^2_{MAI}(e') \end{aligned} \tag{36}$$

The above variances can be computed by taking the second derivatives of the corresponding characteristic functions ((29) and (32)) at $z=1$ [21] which result in

$$\begin{aligned} \sigma^2_{MAI}(e) &= \left(A^{k,q}(e)\right)^2 m_p^2 N_s\beta(\alpha+N_s\beta);\\ \sigma^2_{MAI}(e') &= \left(A^{k,q}(e')\right)^2 m_p^2 N_s\beta\alpha \end{aligned} \tag{37}$$

where $A^{k,q}(e)$ $(A^{k,q}(e'))$ is the amplitude of qth service of user k belonging to the set $E(E')$. The total variance of the interfering users conditioned on U noting (36) is

$$\begin{aligned} \sigma^2_{MAI|U} =& m_p^2 N_s\beta(N_s\beta+\alpha)\sum_{e\in E}\left(A^{k,q}(e)\right)^2\\ &+m_p^2N_s\beta\alpha\sum_{e'\in E'}\left(A^{k,q}(e')\right)^2 \end{aligned} \tag{38}$$

If we assume that the services have the same amplitude, i.e. $A^{k,q}=A,\ k=2,\ldots,K,\ q=1,\ldots,Q$, then

$$\sigma^2_{MAI|U}=A^2m_p^2N_s\beta\{UN_s\beta+(K-1)Q\alpha\} \tag{39}$$

3.1.4 The variance of output noise: The output of the receiver owing to the input noise is

$$n_0=\sum_{j=0}^{N_s-1}n_j \tag{40}$$

where n_j is the noise component in frame j obtained by replacing $n(t)$ instead of $r(t)$ in (9). Now, we find the correlation between noise components at different frames. Considering $E[n(t)n(t')] = N_0/2\delta(t-t')$, and the duration of $w_{rec}(t)$, we can easily obtain

$$E(n_j n_{j'}) = 0 \quad j \neq j'$$

$$E(n_j^2) = \frac{N_0}{2}\int_{jT_f}^{(j+1)T_f}\left\{w_{rec}\left[t - jT_f - c_j^1 T_c - \left(\frac{h_j^{1,1}+1}{2}\right)\delta\right] - w_{rec}\left[t - jT_f - c_j^1 T_c - \left(\frac{-h_j^{1,1}+1}{2}\right)\delta\right]\right\}^2 dt \tag{41}$$

The above integral for $h_j^{1,1} = \pm 1$ considering (14) can be computed as

$$E(n_j^2) = \frac{N_0}{2}\int_0^{T_W+\delta}\{w_{rec}[t] - w_{rec}[t-\delta]\}^2 dt = N_0 m_p \tag{42}$$

Therefore, we obtain

$$E(n_0^2) = \sum_{j=0}^{N_s-1} E(n_j^2) = m_p N_s N_0 \tag{43}$$

3.1.5 Error probability:

The output of the receiver can be written as $Y = \pm A^{1,1} m_p N_s + MAI + n_0$ (the sign $\pm$ depends on the sign of desired service data). We assume Gaussian distribution for the interference based on central limit theorem (CLT) which is accurate for the moderate or high number of users. Also it was shown in [3] that for large number of frames ($N_s > 32$) or users, the exact probability density function approaches to the Gaussian distribution. As the channel is symmetric, the unconditional bit error probability is equal to the conditional bit error probability conditioned on the transmitted data being equal to 1. So the error probability conditioned on U (the cardinality of set E defined before) is easily calculated as

$$P_{e|U} = P_{e|U, D_i^{1,1}=1} = p(MAI + n_0 < -A^{1,1} m_p N_s) = Q\left\{\sqrt{\frac{(A^{1,1})^2 m_p N_s}{A^2 \beta m_p\{U N_s \beta + (K-1)Q\alpha\} + N_0}}\right\} \tag{44}$$

where $Q(x) = 1/\sqrt{2\pi}\int_x^\infty e^{-u^2/2} du$ [22]. The unconditional bit error probability can be obtained as

$$P_e = E_U(P_{e|U}) \tag{45}$$

Now, we compute the probability of U (the number of different users having the same Walsh code as the service 1 of user 1). We assume that the Walsh codes are assigned to the services in a cyclic fashion [23]. In other words, noting that there are totally N_s distinct Walsh codes used in the system, if $m = kQ + q$ is a counting index of the services of the system, then the services with indices m_0 and $m_0 + N_s$ have the same Walsh codes, and any N_s services with sequential indices have distinct Walsh codes. By defining $r = \text{mod}\{KQ, N_s\}$, in which $\text{mod}\{a, b\}$ denotes the reminder of division a to b, we consider three cases:

a) $N_s \geq KQ$ implies that $U = 0$ and $P_e = P_{e|U=0}$.

b) $N_s > KQ$ and $r = 0$ implies that $U = [KQ/N_s] - 1$ and $P_e = P_{e|[U=KQ/N_s]-1}$.

c) $N_s < KQ$ and $r \neq 0$, in this case we have

$$U = \begin{cases} U_1 = \left\lceil\frac{KQ}{N_s}\right\rceil - 1 & p(U_1) = 1 - \frac{r}{N_s} \\ U_2 = \left\lceil\frac{KQ}{N_s}\right\rceil & p(U_2) = \frac{r}{N_s} \end{cases}$$

So, we have

$$P_e = \left(1 - \frac{r}{N_s}\right)P_{e|U_1} + \frac{r}{N_s} p_{e|U_2} \tag{46}$$

3.2 Parallel mapping structure

In this model, we use distinct PN codes for different services. It is easily verified that the output of the system owing to the desired service and the variance of output noise are as (15), (16), and (43), respectively. Since the services of each user are treated as virtual users, therefore we have $KQ - 1$ interfering users. This method can be considered as the special case of the multicode method in which, there is only one service for each user (i.e., no self interference) and all users have the same Walsh code with the desired service, i.e., $U = KQ - 1$. So considering (29) and assuming the independence of users we have

$$\varphi_I(z) = \left\{\frac{1}{2}\left(\alpha + \beta z^{-Am_p}\right)^{N_s} + \frac{1}{2}\left(\alpha + \beta z^{Am_p}\right)^{N_s}\right\}^{KQ-1} \tag{47}$$

and

$$\sigma_{MAI}^2 = (KQ-1)A^2 m_p^2 N_s \beta(N_s\beta + \alpha) \tag{48}$$

Also, note that MAI has zero mean. Therefore, the bit error probability is easily obtained as

$$P_e = Q\left(\sqrt{\frac{(A^{1,1})^2 m_p N_s}{(KQ-1)A^2 m_p \beta(N_s\beta + \alpha) + N_0}}\right) \tag{49}$$

The above result quite agrees with that of obtained in [3] for the case of $N_0 = 0$.

3.3 Serial mapping method

In this method, the duration of one bit is assigned to Q services, so if we consider the same rate for all services, then the number of frames for each service is N_s/Q. As mentioned before for the case that different services have different rates, the number of assigned frames to each service will be variable depending on the rate. In the following, we assume that different services have the same rate. The results can be derived easily for the generalised case. The output owing to the desired service noting (15) and (16) is

$$S/(d_j^{1,1} = 1) = B^{1,1} m_p N_s/Q; \quad S/(d_j^{1,1} = -1) = -B^{1,1} m_p N_s/Q \tag{50}$$

where $B^{1,1}$ is the amplitude in this structure. The output noise variance from (42) and (43) is obtained as

$$E(n_0^2) = \sum_{j=0}^{N_s/Q-1} E(n_j^2) = m_p N_0 N_s/Q \tag{51}$$

The services of each user transmit their data in distinct frames, so there is no self interference. Also only one service of each interfering user overlaps with the desired service. The variance of interference is obtained as (48) by replacing

N_s/Q and $(K-1)$ instead of N_s and $(KQ-1)$, respectively.

$$\sigma^2_{MAI} = (K-1)B^2 m_p^2 \frac{N_s}{Q}\beta\left(\frac{N_s}{Q}\beta + \alpha\right) \quad (52)$$

Therefore the error probability is obtained as

$$P_e = Q\left(\sqrt{\frac{(B^{1,1})^2 m_p N_s}{(K-1)B^2 m_p \beta(N_s\beta + \alpha Q) + N_0 Q}}\right) \quad (53)$$

3.4 Performance comparison

In this part, we compare the performance of the three structures analytically. For a fair comparison, it is necessary that the three systems have the same SNR which is defined as the energy of the desired signal to the variance of noise in one bit duration. So we have

$$SNR_{parallel} = SNR_{multicode} = \frac{(A^{1,1})^2 m_p N_s}{N_0}; \quad SNR_{serial} = \frac{(B^{1,1})^2 m_p N_s}{N_0 Q} \quad (54)$$

By setting the above two relations equal to each other, we obtain

$$(B^{1,1})^2 = Q(A^{1,1})^2 \quad (55)$$

In fact, since the number of frames assigned to each service in the serial mapping method is $1/Q$ times that of the other two methods, in order to have the same transmitted powers for all the three methods, the power of each transmitted pulse in the serial mapping method has to be Q times that of the other methods. We assume a perfect power controlled system, i.e., $A^{1,1} = A$, $B^{1,1} = B$, then by dividing the nominator and denominators of (44), (49) and (53) to A^2 or B^2, it suffices to compare the denominators of the three equations.

a) By comparing the denominators of (49) and (53) (after being divided by A^2 and B^2), we obtain

$$\begin{aligned} &m_p\beta(KQ-1)(N_s\beta + \alpha) + N_0/A^2 \\ &\quad > m_p\beta(K-1)(N_s\beta + \alpha Q) + N_0 Q/B^2 \\ &\quad \Rightarrow (Q-1)(K\beta N_s + \alpha) > 0 \end{aligned} \quad (56)$$

The above inequality always holds true, so we can conclude that the serial mapping method has always better performance than the parallel mapping structure.

b) We compare the serial mapping and multicode methods. Considering $N_s > KQ$ and $U = 0$ in (45), we have

$$P_{e|U=0} = Q\left\{\sqrt{\frac{m_p N_s}{m_p\beta(K-1)Q\alpha + N_0/A^2}}\right\} \quad (57)$$

Comparing the denominators of the above equation and (53) gives

$$\begin{aligned} &m_p\beta(N_s\beta + \alpha Q)(K-1) + N_0 Q/B^2 \\ &\quad > m_p\beta(K-1)Q\alpha + N_0/A^2 \\ &\quad \Rightarrow N_s(K-1) > 0 \end{aligned} \quad (58)$$

The above inequality always holds true. Also for the case of $N_s < KQ$ and $U = [KQ/N_s] - 1$, we obtain

$$P_{e|U=[\frac{KQ}{N_s}]-1} = Q\left\{\sqrt{\frac{m_p N_s}{m_p\beta\{(KQ-N_s)\beta + (K-1)Q\alpha\} + N_0/A^2}}\right\} \quad (59)$$

If we compare the denominators of (53) and (59) we obtain

$$\begin{aligned} &m_p\beta\{(N_s\beta + \alpha Q)(K-1)\} + N_0 Q/B^2 \\ &\quad > m_p\beta\{(KQ-N_s)\beta + (K-1)Q\alpha\} + N_0/A^2 \\ &\quad \Rightarrow N_s > Q \end{aligned} \quad (60)$$

The above condition $(N_s > Q)$ always exists in both the serial mapping and multicode methods. Therefore, the multicode using Walsh subcode method gives always better performance than the serial mapping structure. For the case of $N_s < KQ$ and $r \neq 0$, we will obtain the same result using numerical calculations presented in Section 5.

c) It is easily verified from the above two parts that the multicode using the Walsh subcode method also outperforms the parallel mapping structure.

4 Performance evaluation of coded schemes

In this Section, we first present a brief description of a super-orthogonal coded system, then we evaluate the performance of the three coded multirate structures.

4.1 Coded system

From (2), we can consider an uncoded TH-UWB system as a coded system, in which a simple repetition block code with rate $1//N_s$ is used. To improve the system performance, in [3] applying a near optimal low-rate convolutional code called super-orthogonal code instead of the above simple repetition code is proposed. A super-orthogonal code [13] with constraint length M, has rate $1/2^{M-2}$. Since the rate $1/N_s$ is required, the constraint length of the code must be equal to $M = 2 + \log_2 N_s$. Now, for a coded scheme, d_j^k in (2) is the jth coded symbol corresponding to the current input bit of the code. That is, the location of each pulse in each frame is determined by the user dedicated PN sequence along with the code symbol corresponding to that frame. Note that the coding scheme described does not require any bandwidth expansion compared to the uncoded TH–UWB scheme.

For a convolutional code only the upper and lower bounds on the bit error rate (BER) using a maximum likelihood (ML) decoder (through a Viterbi algorithm) are available. For this purpose, the path generating function of code is required. This function for a super-orthogonal code is given as [13]

$$T_{SO}(\gamma, \mu) = \frac{\mu W^{M+2}(1-W)}{1 - W[1 + \mu(1 + W^{M-3} - 2W^{M-2})]} \quad (61)$$

where $W = \gamma^{2^{M-3}}$. Expanding the above expression, we get a polynomial in terms of γ and μ. The free distance of the code is obtained from the first term of the expansion as

$$d_f = 2^{M-3}(M+2) = N_s(\log_2 N_s + 4)/2 \quad (62)$$

If we consider the uncoded system as a coded scheme with a repetition code, its free distance will be N_s. Comparing the free distance of the two schemes, it is clear that the super-orthogonal coded scheme outperforms the repetition coded scheme (uncoded scheme) significantly.

An upper bound on the BER for a memoryless channel is obtained using union bound as follows [13]

$$P_b < \left.\frac{dT_{SO}(\gamma,\mu)}{d\mu}\right|_{\mu=1} = \frac{W^{M+2}}{(1-2W)^2}\left(\frac{1-W}{1-W^{M-2}}\right)^2 \quad (63)$$

where $W = Z^{2^{M-3}}$. The parameter Z is calculated from the Bhattacharyya bound [13] as

$$Z = \int_{-\infty}^{+\infty} \sqrt{p_{-1}(y)p_1(y)}dy \quad (64)$$

where $p_1(y)$ and $p_{-1}(y)$ are the probability density functions of the output of the correlator in each frame conditioned on the input symbol being 1 and -1, respectively.

A lower bound on the probability of bit error rate is obtained by considering only the first term of path generating function. The result is

$$P_b > P(d_f) \quad (65)$$

where $P(d_f)$, the probability of an error event with Hamming weight d_f, is equal to the probability that the summation of d_f pulse correlator outputs (correlator output in a frame) is less than 0, when the corresponding input symbols (coded bits) are 1.

For a binary input AWGN channel, assuming that η and σ^2 are the mean and variance of the correlator output in each frame, we have [3, 13]

$$Z = \exp\left(-\frac{\eta^2}{2\sigma^2}\right) \quad (66)$$

Using the inequality $Q(\sqrt{x+y}) < Q(\sqrt{x})e^{-y/2}$ [13], we obtain an improved higher bound as

$$P_b < \exp\left(\frac{\eta^2}{2\sigma^2}d_f\right)Q\left(\left(\frac{\eta^2}{\sigma^2}d_f\right)^{1/2}\right)\left.\frac{dT_{SO}(\gamma,\mu)}{d\mu}\right|_{\mu=1}$$
$$= Q\left(\left(\frac{\eta^2}{\sigma^2}d_f\right)^{1/2}\right)\left(\frac{1-W}{(1-2W)(1-W^{M-2})}\right)^2 \quad (67)$$

and the lower bound on BER will be as follows

$$P_b > Q\left(\left(\frac{\eta^2}{\sigma^2}d_f\right)^{1/2}\right) \quad (68)$$

Now, in order to compute the parameters η and σ^2 for each multirate scheme, in the following, we obtain the correlator output in one frame (code symbol duration) owing to the desired signal, self interference, multiple access interference, and noise for each scheme.

4.2 Multicode using Walsh subcode concatenation method

In this method, in which we use two kinds of PN and Walsh codes, since the number of pulses transmitted during each service bit interval is fixed and independent of the number of services, d_f is as (62). The correlator output in frame j owing to the first service of first user (desired service) considering the discussions of Section 3.1.1 is easily obtained as (see (14)–(16))

$$S_j/(d_j^{1,1}=1) = A^{1,1}m_p;$$
$$S_j/(d_j^{1,1}=-1) = -A^{1,1}m_p \quad (69)$$

The self interference in frame j considering (17)–(19) is $SI_j^q = \pm A^{1,q}m_p$, therefore the probability density and characteristic functions are obtained as

$$P_{SI_j^q}(x) = \frac{1}{2}\delta(x - A^{1,q}m_p) + \frac{1}{2}\delta(x + A^{1,q}m_p)$$
$$\Rightarrow \varphi_{SI_j^q}(z) = \frac{1}{2}z^{-A^{1,q}m_p} + \frac{1}{2}z^{A^{1,q}m_p} \quad (70)$$

So the variance of self interference in frame j is computed as

$$\sigma^2_{SI_j^q} = \left.\frac{d^2\varphi_{SI_j^q}(z)}{dz^2}\right|_{z=1} = (A^{1,q})^2m_p^2 \quad \Rightarrow \quad \sigma^2_{SI_j} = m_p^2\sum_{q=2}^{Q}(A^{1,q})^2 \quad (71)$$

The output of the receiver owing to the qth service of the kth user has been obtained in (30), which can take in one of the three values of 0 or m_p or $-m_p$. So, the total interference from all services of user k is

$$I_j^k = \begin{cases} 0 & c_j^k \neq c_j^1 \\ \sum_{q=1}^{Q} I_j^{k,q} & c_j^k = c_j^1 \end{cases}; \quad I_j^{k,q} = \pm A^{k,q}m_p \quad (72)$$

Since the interferences caused by different services of the same user are independent, the probability density function of interference from user k is

$$P_{I_j^k}(x) = \alpha\delta(x) + \beta\left\{\frac{1}{2}\delta(x - A^{k,1}m_p) + \frac{1}{2}\delta(x + A^{k,1}m_p)\right\}$$
$$* \cdots * \left\{\frac{1}{2}\delta(x - A^{k,Q}m_p) + \frac{1}{2}\delta(x + A^{k,Q}m_p)\right\} \quad (73)$$

where $*$ denotes the convolution. The characteristic function is obtained as

$$\varphi_{I_j^k}(z) = \alpha + \beta\left\{\frac{1}{2}z^{A^{k,1}m_p} + \frac{1}{2}z^{-A^{k,1}m_p}\right\}$$
$$\cdots\left\{\frac{1}{2}z^{A^{k,Q}m_p} + \frac{1}{2}z^{-A^{k,Q}m_p}\right\}$$
$$= \alpha + \beta\left\{\frac{1}{2}z^{A^{k,1}m_p} + \frac{1}{2}z^{-A^{k,1}m_p}\right\}^Q \quad (74)$$

The variance of interference assuming the same amplitude for all services is calculated as

$$\sigma^2_{I_j^k} = A^2m_p^2\beta Q \quad (75)$$

We also assume that users are independent, so the variance of the total MAI is computed as

$$\sigma^2_{MAI_j} = (K-1)m_p^2A^2\beta Q \quad (76)$$

The variance of noise in the jth frame is obtained as (42). Therefore the total variance is

$$\sigma^2 = \sigma^2_{MAI_j} + \sigma^2_{SI_j} + \mathrm{var}(n_j)$$
$$= (K-1)Q\beta A^2m_p^2 + (Q-1)m_p^2A^2 + m_pN_0 \quad (77)$$

4.3 Parallel mapping structure

In this case, which we use distinct PN codes to differentiate users and their services alike, like the previous structure, d_f is as (62). The outputs owing to the desired signal and noise are the same as those derived for the previous structure presented in (69) and (42). For computing the variance of the total interference in each frame, note that the number of independent interfering services is $KQ-1$ (including the interferences from the other services of the desired user). The characteristic function of interference in one frame owing to each interfering service conditioned on input data is as (27) or (30). Therefore, the unconditional characteristic function and the corresponding variance are easily computed as

$$\varphi_{I_j^{k,q}}(z) = \alpha + \frac{\beta}{2} z^{-A^{k,q} m_p} + \frac{\beta}{2} z^{A^{k,q} m_p}$$
$$\Rightarrow \sigma^2_{I_j^{k,q}} = (A^{k,q})^2 m_p^2 \beta \quad (78)$$

and if we assume the same amplitude for all interfering users, then the total variance is

$$\sigma^2 = (KQ - 1)\beta m_p^2 A^2 + m_p N_0 \quad (79)$$

4.4 Serial mapping method

In this case, since the number of pulses transmitted during each service interval is equal to N_s/Q, the code rate is Q/N_s, and therefore from (62) the free distance will be

$$d_f = \frac{N_s}{Q}\left[\log_2\left(\frac{N_s}{Q}\right) + 4\right] \Big/ 2 \quad (80)$$

The output owing to the desired service signal is the same as the other two methods (69), and the variance of noise is as (42). There is no self interference, and the number of interfering users is $K-1$. The variance of each interfering user is the same as (78). Hence the total variance is

$$\sigma^2 = (K - 1)\beta m_p^2 B^2 + m_p N_0 \quad (81)$$

4.5 Performance comparison

The upper and lower bounds on the bit error rate (BER) for each method are obtained as in (67) and (68) by using $\eta = A^{1,1} m_p$ or $\eta = B^{1,1} m_p$ where $(B^{1,1})^2 = Q(A^{1,1})^2$ and replacing (77), (79) or (81) instead of σ^2. Also, we use the corresponding value of d_f for different structures. Note that for serial mapping method $W = Z^{N_s/2Q}$ and for the other two methods $W = Z^{N_s/2}$ where $Z = \exp(-(A^{1,1})^2 m_p^2/2\sigma^2)$ or $Z = \exp(-(B^{1,1})^2 m_p^2/2\sigma^2)$.

a) We compare parallel and serial mapping methods. Comparison of (79) and (81) shows that σ^2/η^2 (normalised variance) of the serial mapping method is less than that of the parallel mapping structure. But, d_f (62) of the parallel mapping method is larger than that of the serial mapping method (80) by a factor more than Q, which is more than the ratio of the σ^2/η^2 of the two methods. Therefore, we can conclude that the lower bound on the BER (68) of the serial mapping method is higher than that of the parallel mapping structure. We obtain the same result for the upper bound of BER (67) using numerical calculation. We will also find that the lower bound of serial mapping method is higher than the upper bound of parallel mapping method. Since the bit error rate lies between the lower and upper bounds, we conclude that the parallel mapping method always outperforms the serial mapping method.

b) It is seen from (77) and (79) that the variance of the parallel mapping method is less than the multicode method. This can also be justified as follows. In the case of multicode using the Walsh subcode method, there is self interference in each frame with probability 1 (see (70)), in contrast to the parallel mapping case in which the other services are treated as MAI so that the other services of the same user can interfere with probability of β as in MAI (see (78)). Since the two structures have the same d_f (62) and η, considering (68) we can conclude that the lower bound of the bit error rate in the multicode using the Walsh subcode method is higher than that of the parallel mapping method. The same result is obtained for the upper bounds of the two methods.

c) We observe from (77) and (81) that the multicode method has higher σ^2/η^2 and d_f than the serial mapping method. However, the ratio of σ^2/η^2 of these two methods may be less or larger than the ratio of their d_fs. So, the parameters of the system such as N_s and K indicate which of these two methods (serial mapping or multicode methods) has better performance. We will consider and compare these two methods using numerical examples.

5 Numerical results

In this Section, we evaluate the performance of the three aforementioned multiservice-multirate methods using numerical results. The received pulse is modelled as $w_{rec}(t + T_w/2) = [1 - 4\pi(t/\tau_m)^2]\exp[-2\pi(t/\tau_m)^2]$ [2] where $\tau_m = 0.2877$ ns, $T_w = 0.7$ ns, $\delta = 0.156$ ns, and $T_f = 100$ ns, so the parameter m_p will be equal to 1.75×10^{-10}. It is worth mentioning that in the performance analysis, we have computed the bit error rate for the general case of pulse shapes without considering any special pulse. The effect of pulse shape appears through the parameter $m_p \triangleq \int_0^{T_w+\delta} w_{rec}(t)[w_{rec}(t) - w_{rec}(t-\delta)dt]$ which affects only on the SNR. So, the computations and results throughout the paper are independent from the pulse shapes. Only in the numerical results have we considered the Gaussian pulse which is commonly used in the literature. The signal-to-noise ratio (SNR) is defined as $\text{SNR} = (A^{1,1})^2 m_p N_s/N_0$ or $\text{SNR} = (B^{1,1})^2 m_p N_s/N_0 Q$ (54). In order to have fair comparison, the three systems must have the same parameters (N_s, N_h, K, Q) and SNR, which implies $(B^{1,1})^2 = Q(A^{1,1})^2$. We also assume a perfect power controlled system, i.e. $A^{1,1} = A$, $B^{1,1} = B$.

We first consider the uncoded scheme. Figure 2 shows the bit error rate of the three multirate structures against the SNR. It is observed that the multicode using Walsh subcode method outperforms the other two methods, while the serial mapping structure works better than the parallel mapping method. The reason is that the self interference in the multicode method is zero in one bit duration (N_s frames) because of the orthogonlity of the Walsh codes (19), while in the parallel mapping method the effect of self interference appears with the probability β, the same as the multiple

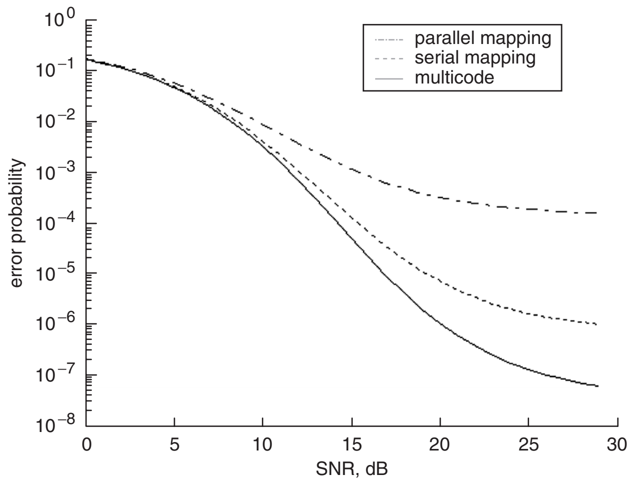

Fig. 2 *Performance of the three methods against SNR in the uncoded scheme*
$K = 10$, $Q = 4$, $N_h = 32$, $N_s = 32$

access interference. Even though in the serial mapping method, there is also no self interference (as the services send their data in distinct frames), but the number of frames assigned to each service is Q times less than that of the multicode method, which makes its performance be worse than the multicode method. The results obtained quite agree with the discussions in Section 3.4.

Note that in the performance analysis, we have considered the average bit error rate ((45), $P_e = E_U\{P_{e|U}\}$), i.e. the bit error rate owing to all kinds of interfering services. When $N_s < KQ$, as in Fig. 2, some services of different users have the same Walsh codes as the desired service while the others have different Walsh codes. We considered the effect of both kinds of services when obtaining the variance of interference (33)–(39).

In Fig. 3, the effect of the number of users on the performance of the system in the uncoded scheme is demonstrated. Again, we observe that the multicode using Walsh subcode method gives the best performance compared to the other ones.

We have examined the performance of the system against the number of services. The results are illustrated in Fig. 4. Again, the multicode method in the uncoded scheme has the best performance among the methods considered.

We have also evaluated the performance of coded schemes for the three methods. Figures 5 and 6 show the bit error rate of the uncoded schemes and the upper and lower bounds on the bit error rate of the coded schemes against the number of users. We observe that for all three methods, the coded schemes outperform the uncoded schemes significantly while there is no need for extra bandwidth in the coded scheme compared to the uncoded one.

It is observed from the Figs. 5 and 6 that the lower and upper bounds on BER of the parallel mapping method are less than those of the other two methods, respectively. However, the lower bound of serial mapping method is higher than the upper bound of parallel mapping method. Since the error rate lies between the lower and upper bounds, so we can conclude that the parallel mapping method outperforms the serial mapping method.

It is also seen that for low error rates (low number of users, i.e., higher signal-to-interference-plus-noise (SINR) ratios), the lower and upper bounds on BER of the three

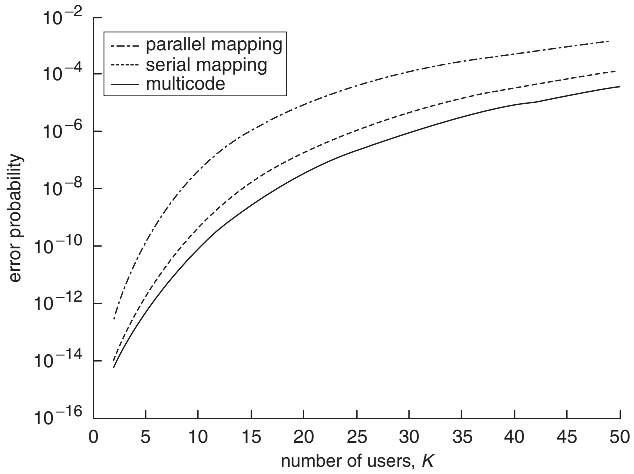

Fig. 3 *Performance of the three methods against number of users in the uncoded scheme*
SNR = 18 dB, $Q = 4$, $N_h = 64$, $N_s = 64$

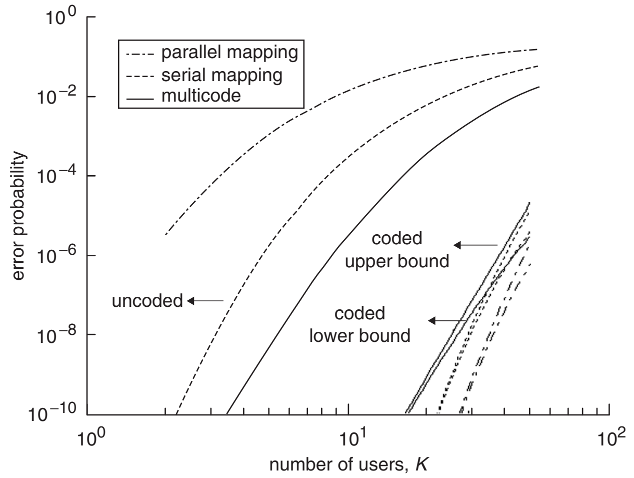

Fig. 5 *Performance of the three methods against number of users in uncoded and coded schemes*
SNR = 18 dB, $Q = 4$, $N_s = 64$, $N_h = 16$

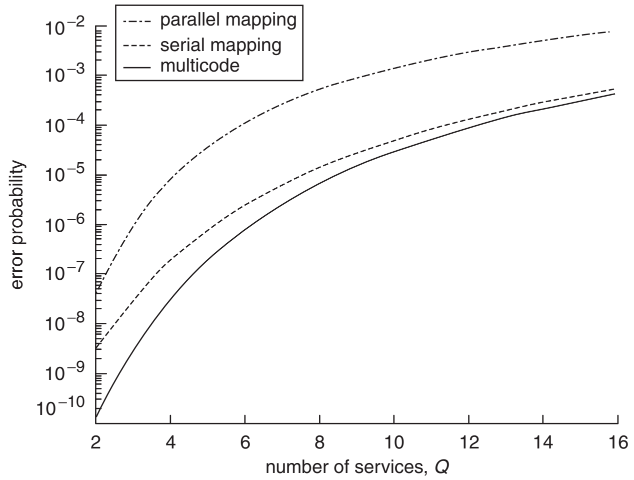

Fig. 4 *Performance of the three methods against number of services in the uncoded scheme*
SNR = 18 dB, $K = 20$, $N_h = 64$, $N_s = 64$

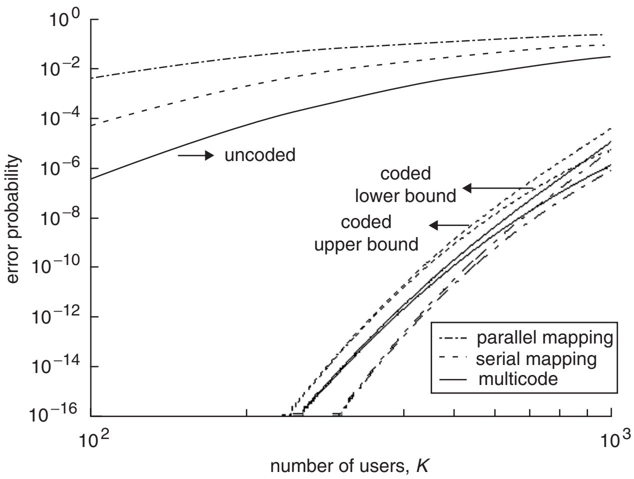

Fig. 6 *Performance of the three methods against number of users in uncoded and coded schemes*
SNR = 18 dB, $Q = 4$, $N_s = 256$, $N_h = 64$

methods are very close but they are separated when the number of users increases (lower SINR). However, when the two bounds are very close the parallel mapping method has the best performance.

In Fig. 5, the parallel mapping method works the best, in general. At low number of users, the serial mapping method has better performance than the multicode method, but at high number of users, the two methods have almost the same performance. In Fig. 6, in which larger value of N_s is applied compared to Fig. 5, the serial mapping method has the worst performance. The discussions of Section 4.5 confirm the above results.

The great difference in the performance of uncoded and coded schemes is that the parallel mapping has the worst performance in the uncoded scheme while it works the best in the coded scheme. This phenomenon can be justified as follows. In the multicode method, the self interference (SI) in each frame has the nonzero value $\pm Am_p$, while it has the total value of zero in one bit duration (N_s frames) because of the orthogonal property of the Walsh code. However, in the parallel mapping method, the self interference can be treated as a virtual user and therefore its effect is the same as MAI. As a result, in the uncoded scheme the total interference of the multicode method is less than that of the parallel method, which implies a better performance for the multicode method compared with the parallel mapping method. In the coded scheme, however, the effect of self interference appears with probability 1 for the multicode method while for the parallel method it appears with probability β. So the total interference of multicode method is more than that of the parallel method which makes the performance of multicode method be worse than the parallel one in the coded scheme.

Alternatively, the serial mapping method has zero self interference both in uncoded and coded schemes because the services are separated in time. In addition, only one service of each interfering user overlaps with the desired user. As a result, the serial mapping method has less interference than the parallel mapping method in both coded and uncoded schemes, which implies better performance for this method compared to the parallel mapping method in the uncoded scheme. However in the coded scheme, the free distance is also an important factor. The free distance of the parallel mapping method is larger than that of the serial mapping method by a factor more than Q, which is more than the ratio of the normalised interference variance of the two methods. Hence, the serial mapping method has less performance than the parallel mapping method in the coded scheme.

Although the multicode and serial mapping methods have zero self interference in the uncoded scheme, the MAI in the multicode is less than that of the serial mapping which results in the best performance of the multicode method in the uncoded scheme. In the coded scheme, the serial mapping has less variance than the multicode (as the self interference is zero in the serial mapping method while it is not zero in the multicode mapping method and also only one service of each interfering user affects on the interference variance). Alternatively, the serial mapping has less free distance (less coding gain). But the ratio of the normalised variances of the two methods is not the same as the ratio of their free distances, so the system parameters determine which method has better performance. For example, at large values of N_s, it is expected that the multicode method performs better than the serial mapping method, as the coding gain effect is higher. The numerical results presented in Figs. 5 and 6 verify this expectation.

6 Conclusions

In this paper, we considered three methods for multirate multiservice TH-UWB systems. In the parallel mapping method, one kind of PN signature code is used to distinguish both services and users. In the serial mapping method, the services of each user have the same PN code but they send their information in distinct frames. In the multicode structure, PN codes are assigned for distinguishing the users and Walsh codes for differentiating the services of the same user.

We evaluated the performance of the three methods in uncoded and coded schemes over an AWGN channel using the conventional correlator receiver. For the uncoded scheme, we have observed that the multicode method has the best performance, and it has also been realised that the serial mapping method performs better than the parallel mapping method. However, for the coded scheme, the parallel mapping method has shown the best performance and the multicode method works better than the serial mapping for larger N_s. We also compared the coded scheme with the uncoded scheme. It was shown that the coded schemes of the three methods significantly outperform the uncoded schemes without requiring any extra bandwidth.

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