Statistical Modeling of Consecutive Range Profiles for Radar Target Recognition

Abdollah Ajorloo*, Mahdi Hadavi*, Mohammad Mahdi Nayebi*, and Mohammad Hassan Bastani*

*Electrical Engineering Department, Sharif University of Technology
Azadi Street, Tehran, IRAN
Ajorloo@ee.sharif.edu

Abstract: *The high resolution range profile (HRRP) is known as the most important tool in radar target recognition. Mainly, the sensitivity to the aspect angle limits the performance of the related methods. To overcome this problem, Gaussian mixture distribution is used to model the short-term relation of consecutive HRRPs. In this work, an alternative dynamical system based method is proposed to overcome the limitations of recent methods in the field such as the independency assumption. Here, the performance of the method is tested by the data produced in an electromagnetic simulation for the radar return from an aerial maneuvering target. The results show the performance of the proposed method comparing to Gaussian mixture and factor analysis based methods using the Akaike information criterion (AIC).*

1. Introduction

Automatic target recognition (ATR) is widely considered using high resolution radar capabilities. The HRRP has been known as the most powerful tool in the field, [1-4]. The main limitation of recognition methods based on HRRP is the sensitivity to the target aspect angle from radar, [2], [5]. The moving toward range cells (MTRC) and speckle are the most important phenomena causing this sensitivity, [2].

To overcome this problem and using the information in a frame of range profiles in the recognition process, we need a mathematical model for the statistical relation of the consecutive range profiles. Some solutions are proposed using the Gaussian distribution and its variations in [6] and [7]. In [6] the features extracted from the range profiles are modeled by Gaussian mixture distribution. Note that in all of these works, the consecutive range profiles in a frame or segment of range profiles are assumed to be statistically independent from each other with the same distribution (IID).

Here we seek for an alternative model for the problem ignoring the independency assumption. According to the physical behavior of linear and rotational movement of the target and electromagnetic backscattering theory, it could be concluded that a linear dynamical system can be used to model the statistical behavior of dominant peak locations in consecutive range profiles.

The model considered here is the general form of the dynamical system with measurement and model noise ignoring the system inputs. To evaluate the fitness of proposed model, we use the simulated data produced from a real maneuver including a reliable electromagnetic numerical solution. The Akaike information criteria (AIC), introduced in [8], is used to compare the dynamical system based model with Gaussian mixture model (GMM). It is shown to be well performed in multivariate model selection problems with limited observation data, [9]. We also, include the results for factor analysis (FA) model which is considered as an alternative for Gaussian mixture model [7],[10].

2. Problem Formulation and Assumptions

According to electromagnetic backscattering theory we can approximate the return of a radar employing a linear frequency modulated (LFM) signal, from an extended target by the following relation:

$$x_n = \sum_{k=1}^{K} \alpha_k \, exp\left(j2\pi f_k n\right) + e_n \qquad (1)$$

Where α_k and f_k denote the complex amplitude and frequency of the k-th backscattering point and K is the number of them. The time index is denoted by $n = 0,1,\dots,N-1$ where N is the number of the range profile samples e_n is white noise or AR noise .

Denoting the LFM rate by μ, we have:

$$f_k = \frac{\mu(R_k\text{-}R_0)}{c} \qquad (2)$$

Where R_k is the k-th scattering point range and R_0 denotes the reference range of the target estimated in the track phase. Here we choose the related frequencies of dominant scattering points as a feature vector for the recognition algorithms. To extract the feature vector we use the RELAX algorithm introduced in [11]. In the following chapters we denote the feature vector of the t- th range profileby y_t and we denote the segment of observations by Y.

3. Dynamical System Model

The dynamical system model used here can be summarized in state update and measurement update equations below:

$$x_{t+1} = Fx_t + w_t \qquad (6)$$

$$y_t = Hx_t + v_t \qquad (7)$$

Where x_t is the hidden state, y_t is the observation and w_t and v_t are the model and measurement noise respectively. Model and measurement noise are assumed to be white Gaussian and uncorrelated from each other.

$$w_t \sim N\left(\mu_w, Q\right) \qquad (8)$$

$$v_t \sim N\left(\mu_v, R\right) \qquad (9)$$

The initial state (x_0) is assumed to be Gaussian too. The parameters of the model can be summarized in the parameter set λ:

$$\lambda = \left(F, H, \mu_w, Q, \mu_v, R\right) \qquad (10)$$

To estimate the parameters an Expectation Maximization (EM) based technique is used which is first introduced in [12]. Using the EM algorithm for estimating the parameters of the dynamical system model involves computing the conditional expectations of the sufficient statistics for the hidden state during the E-step, using these to re-estimate the parameters during the M-step, and iterating until convergence. If Y = [y_0, y_1, …., y_N] be the segment of observations for training the dynamical system, only the following statistics are needed to be computed in E-step [12]

$$E\{x_t|Y\} = \hat{x}_{t|N} \qquad (11)$$

$$E\{x_t x_t^T|Y\} = \hat{x}_{t|N}\hat{x}_{t|N}^T + \Sigma_{t|N} \qquad (12)$$

$$E\{x_t x_{t-1}^T|Y\} = \hat{x}_{t|N}\hat{x}_{t-1|N}^T + \Sigma_{t,t-1|N} \qquad (13)$$

These statistics are calculated using the fixed-interval smoothing form of the Kalman filter, including forward and backward recursions as shown below, augmented with cross-covariance recursions to get second-order statistics.
Assuming $\mu_v = 0$, we have:

Forward recursion:

$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + K_t e_t \tag{14}$$
$$\hat{x}_{t+1|t} = F\hat{x}_{t|t} + \mu_w \tag{15}$$
$$e_t = y_t - H\hat{x}_{t|t-1} \tag{16}$$
$$K_t = \Sigma_{t|t-1} H^T \Sigma_{e_t}^{-1} \tag{17}$$
$$\Sigma_{e_t} = H\Sigma_{t|t-1} H^T + R \tag{18}$$
$$\Sigma_{t|t} = \Sigma_{t|t-1} - K_t \Sigma_{e_t} K_t^T \tag{19}$$
$$\Sigma_{t,t-1|t} = (I - K_t H) F \Sigma_{t-1|t-1} \tag{20}$$
$$\Sigma_{t+1|t} = F\Sigma_{t|t} F^T + Q \tag{21}$$

Backward recursion:

$$\hat{x}_{t-1|N} = \hat{x}_{t-1|t-1} + A_t[\hat{x}_{t|N} - \hat{x}_{t|t-1}] \tag{22}$$
$$\Sigma_{t-1|N} = \Sigma_{t-1|t-1} + A_t\left[\Sigma_{t|N} - \Sigma_{t|t-1}\right] A_t^T \tag{23}$$
$$A_t = \Sigma_{t-1|t-1} F_{t-1}^T \Sigma_{t|t-1}^{-1} \tag{24}$$
$$\Sigma_{t,t-1|N} = \Sigma_{t,t-1|t} + \left[\Sigma_{t|N} - \Sigma_{t|t}\right] \Sigma_{t|t}^{-1} \Sigma_{t,t-1|t} \tag{25}$$

After calculation of (11), (12) ,and (13) in E-step, we must re-estimate the model parameters in M-step, Let us, define the following operators

$$<o>_1 = \frac{1}{N+1} \Sigma_{t=0}^{N} o \tag{26}$$
$$<o>_2 = \frac{1}{N} \Sigma_{t=1}^{N} o \tag{27}$$

Then, in M-step we estimate the parameters as follows

$$\begin{bmatrix} \hat{F} & \hat{\mu}_w \end{bmatrix} = \langle \begin{bmatrix} E\{x_{t+1}x_t^T|Y\} & E\{x_{t+1}|Y\} \end{bmatrix} \rangle_2 \cdot \left(\left\langle \begin{bmatrix} E\{x_t x_t^T|Y\} & E\{x_t|Y\} \\ E\{x_t^T|Y\} & 1 \end{bmatrix} \right\rangle_2 \right)^{-1} \tag{28}$$
$$\hat{Q} = \langle E\{x_{t+1}x_{t+1}^T|Y\} \rangle_2 - \langle \begin{bmatrix} E\{x_{t+1}x_t^T|Y\} & E\{x_{t+1}|Y\} \end{bmatrix} \rangle_2 \cdot \begin{bmatrix} \hat{F} & \hat{\mu}_w \end{bmatrix}^T \tag{29}$$

$$\hat{H} = \langle y_t E\{x_t^T|Y_{a_i}\} \rangle_1 \left(\langle E\{x_t x_t^T|Y\} \rangle_1 \right)^{-1} \tag{30}$$
$$\hat{R} = \langle y_t y_t^T \rangle_1 - \hat{H} \langle E\{x_t|Y\} y_t^T \rangle_1 \tag{31}$$

4. Experimental Results

To evaluate the performance of the models we simulate an aerial maneuvering target with six degree of freedom by JSBSim software. The target is assumed under tracking by a high resolution radar with high bandwidth linear frequency modulated (LFM) transmitted signal without miss observations. To simulate the returned signal from the target we use the simplest component analysis method introduced in [13]. In simulation the effect of noise has been taken into account.

Here the Akaike information criterion is used to evaluate the fitness of the models. If the maximum likelihood of the observed segment of features vectors is denoted by Y and the number of independent parameters in the ith model (M_i) is denoted by p_i , then we have:

$$\mathrm{AIC} = -2log\big(P(Y|M_i)\big)+2p_i \qquad (32)$$

In fig. 1-3 the –AIC is plotted for the three presented models for different observation segments from the maneuver. For each of GMM and FA, the best model that minimizes the AIC has been selected, i.e. the best number of Gaussians in GMM and the best number of factors in FA. In fact, the largest –AIC (smallest AIC) determine the best fitted model. As be seen, in modeling the observations, the dynamical system model outperforms the GMM and FA models.

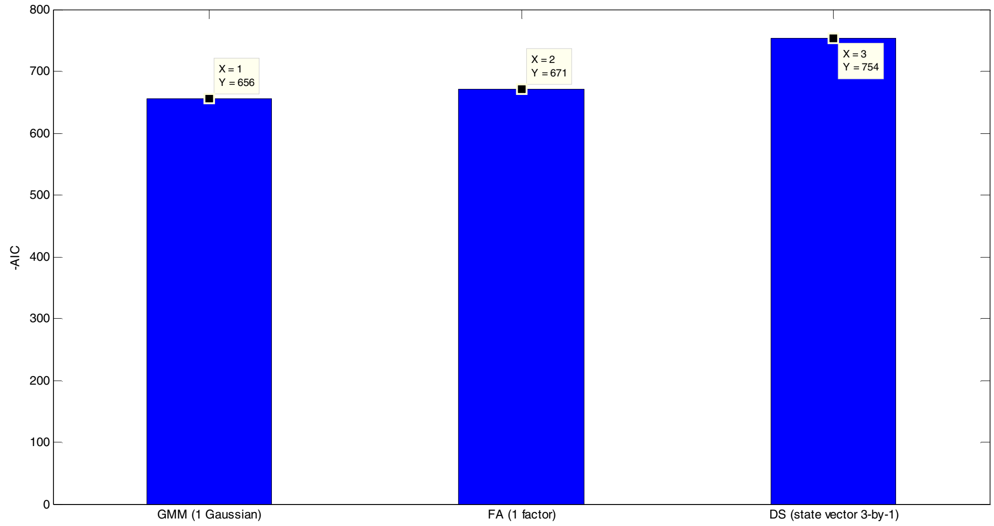

Figure 1. -AIC of the models for a segment of observations consisting of 32 feature vectors.

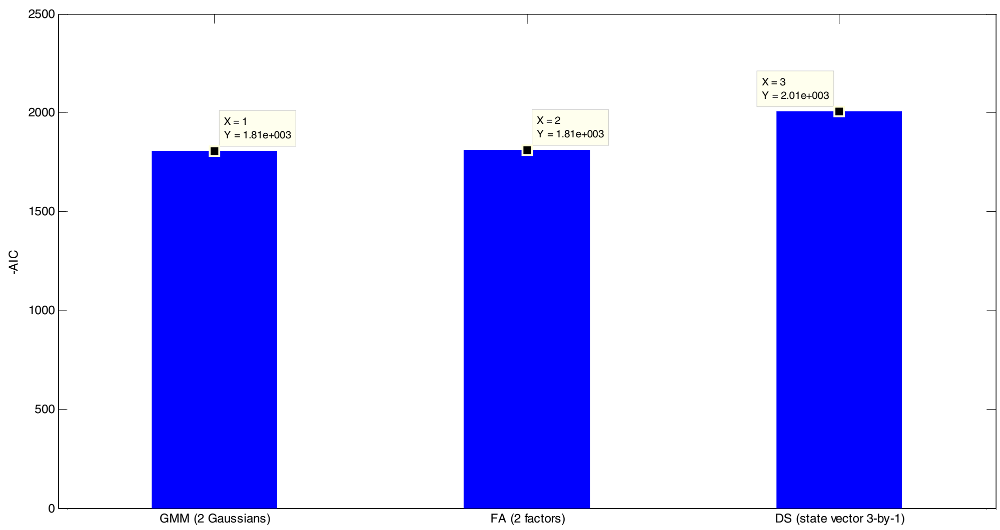

Figure 2. -AIC of the models for a segment of observations consisting of 81 feature vectors.

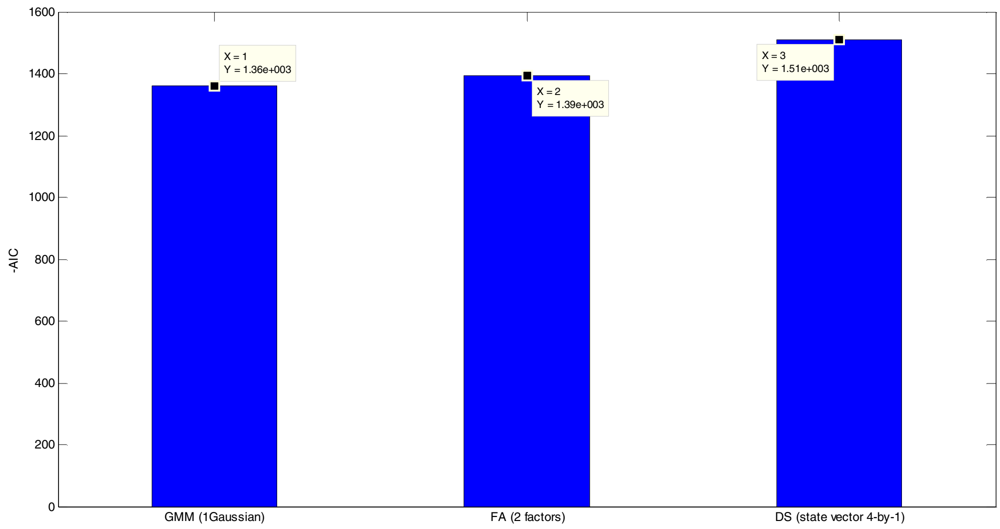

Figure 3. -AIC of the models for a segment of observations consisting of 44 feature vectors.

5. Conclusion

A brief tutorial on the radar HRRP modeling was presented. These models are used to apply consecutive range profiles for radar target recognition tasks. A new modeling approach based on dynamical systems was presented here. The method is shown to be well performed in practical scenarios comparing GMM and FA based methods according to AIC.

References:

[1] H. J. Li, and S. H.Yang, "Using range profiles as feature vectors to identify aerospace objects",*IEEE Trans. Antennas and Propagation*, 1993, Vol. 41, No. 3, pp. 261-268.

[2] S. Hudson, and D. Psaltis, "Correlation filters for aircraft identification from radar range profiles",*IEEE Trans. Aerospace and Electronic Systems*, 1993, Vol. 29, No. 3, pp. 741-748.

[3] R. A. Mitchell, and R. DeWall, "Overview of high range resolution radar target identification",*Proc. Automatic Target Recognition Working Group*, Monterey, CA, 1994.

[4] R. A.Mitchell, and J. J. Westerkamp, "Robust statistical feature based aircraft identification",*IEEE Trans. Aerospace and Electronic Systems*, 1999, Vol. 35, No. 3, pp. 1077-1094.

[5] X. Liao,Z.Bao, and M. Xing, "On the aspect sensitivity of high resolution range profiles and its reduction methods",*IEEE International Radar Conference Record*, 2000, pp. 310-315.

[6] F. Zhu, X. Zhang, Y. Hu, and D. Xie, "Non-stationary hidden Markov models for multi-aspect discriminative feature extraction from radar targets", *IEEE Trans. Signal Processing*, 2007, vol. 55, Iss. 5, pp. 2203–2214.

[7] L. Du, H. Liu, and Z. Bao, "Radar HRRP statistical recognition: parametric model and model selection", *IEEE Trans. Signal Processing*, 2008, Vol. 56, No. 5, pp. 1931-1944.

[8] H. Akaike, and M. Hirotugu, "A new look at the statistical model identification", *IEEE Trans. Automatic Control*, 1974, Vol. 19, No.6, pp. 716–723.

[9] S. Sclove, "Application of model-selection criteria to some problems in multivariate analysis",*Psychometrika*, 1987, Vol. 52, No. 3, pp. 333–343.

[10] H. Akaike, "Factor analysis and AIC", *Psychometrika*,1987, Vol. 52, No. 3, pp. 317–332.

[11] J. Li and P. Stoica, "Efficient mixed-spectrum estimation with applications to target feature extraction",*IEEE Trans. Signal Processing*, 1996, Vol. 44, pp. 281–295.

[12] V. Digalakis, J. R. Rohlicek, M.Ostendorf, “ML Estimation of a Stochastic Linear System with the EM Algorithm and its Application to Speech Recognition”,*IEEE Trans. Speech and Audio Processing*, 1993, Vol. 1, Iss.4, pp. 441-452.

[13] Y. D. Shirman, Computer Simulation of Aerial Target Radar Scattering, Recognition, Detection, and Estimation, Artech House, 2001.