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Spectrally encoded code-division multiple access for cognitive radio networks

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Abstract: Recent studies have revealed that spectrum congestion is due to the inefficient usage of spectrum against the availability. Cognitive radio is viewed as an approach for improving the utilisation of radio spectrum. The spectrally encoded (spread-time) code-division multiple access (SE-CDMA) technique (which is regarded as the dual of spread spectrum CDMA) is considered and its performance in cognitive radio networks is studied. In cognitive network spectrum, overlay and underlay techniques are employed to enable the primary and secondary users to coexist while improving the overall spectrum efficiency. It is shown that SE-CDMA provides considerable flexibility to design overlay and underlay waveforms that are scenario-dependent. The performance of SE-CDMA method is evaluated in additive white Gaussian noise and fading channels in terms of both primary and secondary users. Moreover, its performance is compared with cognitive radio using soft-decision spectrally modulated spectrally encoded waveforms. The results indicate the efficiency of SE-CDMA.

1 Introduction

Spectrum crowding will continue to increase as the demand for higher data rates coupled with growth in new applications and number of users increases. According to the Federal Communications Commission (FCC), large parts of the licensed spectrum resources are unused [1]. To overcome this problem, cognitive radio (CR) has been proposed by allowing an unlicensed user to use a spectrum hole (i.e. a frequency band licensed to a primary user but not utilised), which improves the efficient usage of the spectrum resources [2]. CR is an intelligent network that adapts its internal states with variations in surrounding environment.

The basic idea in CR is to assign the licensed spectrum to the unlicensed users (called secondary users) while having minimum acceptable interference with the licensed users (called primary users). Task spectrum overlay and underlay are two approaches commonly considered to accomplish this. Spectrum overlay allows unlicensed secondary users to utilise unused spectrum simultaneously with primary users. Similarly, spectrum underlay allows unlicensed secondary users to simultaneously operate in primary user bands but under strict transmit power constraints [3, 4]. To achieve this objective in CR, the physical layer needs to be highly flexible and adaptable. The commonly used signalling in a CR is orthogonal frequency domain multiplexing (OFDM). Communication systems based on OFDM have advantages in spectral efficiency but at the cost of being sensitive to the environment impairments such as frequency offset, timing offset, carrier phase noise, multipath issues, Doppler effect and peak to average power ratio. A receiver needs to synchronise with the transmitter in frequency, phase and time. This is not a trivial task especially in a mobile environment where the circumstances vary frequently [5]. To this end, we propose an alternative scheme. Spectrally encoded code-division multiple access (SE-CDMA) technique is one of the newly emerged technologies with high efficiency compared to spread spectrum techniques. SE-CDMA can be a candidate for realising CR concepts and can be employed as a scalable and adaptive technology for CR physical layer.

In recent works [3, 4], the authors studied overlay and underlay CR waveforms using soft-decision spectrally modulated spectrally encoded (SD-SMSE) framework. In their study, depending on the CR user needs, various multi-carrier waveforms can be generated; for example, OFDM, multi-carrier code-division multiple access (MC-CDMA), carrier interferometry (CI) MC-CDMA and transform domain communication system. They used OFDM for primary users and MC-CDMA for secondary users in performance analysis of their system.

The SE-CDMA technique [6–12] is an alternative multiple access scheme to direct-sequence spread spectrum (DS-SS) and is considered as the time–frequency dual of DS-CDMA. As opposed to DS-CDMA which spreads the signal energy in frequency domain, SE-CDMA spreads the energy of the corresponding data pulse in the time domain. In SE-CDMA, the code sequence associated to each user is directly applied in the spectral domain. Using frequency domain, pulse shaping can be easily applied. SE-CDMA is also called spread-time CDMA (ST-CDMA).

The spectrally encoded technique was first introduced in the context of optical CDMA for femto-second and pico-second light pulses [9], then extended to the radio frequency and applied to ultra wideband (UWB) nanosecond pulse systems. The application of SE technique in UWB communications could result in significant performance improvement and has many fundamental and enriching principles and features.

SE-CDMA has several advantages over DS-CDMA such as power efficiency, easier rejection of narrowband interference (NBI), more resistance to fading and the ability to utilise unused frequency bands [6–15]. As another advantage of SE-CDMA, consider a recently proposed multi-access technique based on UWB pulse transmission where we need pulses with the high time resolution. In many applications, transmitting a train of UWB or ultra-short pulses may be undesirable because of their impulsive noise characteristic that may cause degradation in systems operating in contiguous frequency band. Hence, reducing the effective instantaneous power of transmitted UWB pulses may prove to be necessary, and the most viable scheme to do so is based on spread-time technique [7, 8].

The ability of matching the transmitted spectrum with the channel spectrum such as when the channel has support on disconnected frequency bands is one of the attractive features of spectrally encoded technique.

The direct application of SE technique to the transmission channels with disjoint or non-continuous frequency band allocation had been studied in [6, 7]. This feature is an extremely important feature in UWB communication systems especially when it encounters multiple interferences and disjoint spectrum allocation mask, which is typical in most UWB environments.

Additionally, it has been shown that the signal-to-interference ratio in the SE technique is better than the spread spectrum (SS) technique in additive white Gaussian noise (AWGN) and fading channels [6, 7].

The ability of SE-CDMA to coexist with several common radio systems has been analysed for UWB impulse radio [7, 8, 11]. In [11], the authors studied a UWB communication system that uses the SE technique to suppress the NBI via spectral shaping. In [12], the effect of the imperfect channel estimation of UWB systems in the presence of NBI was analysed for SE-CDMA and DS-CDMA systems. Furthermore, in [13], a combined time-hopping (TH) and spectrally encoded (SE) multiple access technique was studied. The results indicate that the combined method has much better performance while it has the same bandwidth and spectral efficiency as the conventional SE-CDMA system. In [14], the power level profile of SE-CDMA signals has been analysed. The performance of the system under multiple Gaussian interferences with Rake and maximum likelihood (ML)-based receivers has been studied in [15, 16], and a three-level coding scheme has been proposed. In [17], the authors designed novel orthogonal spreading codes for SE-CDMA that eliminates multiple access interference (MAI). In [18], the authors analysed the performance of matched and partially-matched receivers for a typical UWB spectrally encoded in the presence of multiple tone jammers. They investigated a scenario where there is no information about the jammer frequencies in the transmitter side while it is desirable to cancel it in the receiver side. The results show that the partially-matched receiver outperforms the conventional-matched filter in case of high jammer to signal power ratio.

The advantages of SE-CDMA are inherent flexibility of spectrum shaping and spectrum management, the ability of adapting to channel conditions such as non-continuous frequency bands and having better performance compared to DS-CDMA. The above characteristics can be an encouragement to employ SE-CDMA in CR systems in order to obtain an effective scheme that improves the overall performance of the system in comparison with commonly used systems.

To the best of the authors' knowledge, so far there has not been any research on the implementation of SE-CDMA in cognitive systems. In this paper, SE-CDMA technique is investigated as a candidate for CR systems. In the following, CR features and requirements are discussed and the ability of SE-CDMA to satisfy these requirements is explained.

- *Spectrum sensing:* CR should be able to identify the unused portions of the spectrum in a fast and efficient way. Inherent frequency domain operation of SE-CDMA can ease spectrum sensing in the frequency domain.
- *Efficient spectrum utilisation (spectrum shaping):* After a CR system scans the spectrum and identifies the primary users and unused parts of spectrum, the cognitive secondary users must adjust their waveforms with these frequency bands. SE-CDMA technique can provide such flexibility. In this method, the waveforms can easily be shaped by simply assigning zero codes to the bands in which the primary users exist.
- *Adaptation and scalability:* SE-CDMA system can be adapted to different transmission environments. Some adaptable parameters are the number of frequency chips, overall bandwidth and chip bandwidth, modulation, codes and signal power. These adaptive parameters can be optimised to achieve various results such as increasing the system throughput, reducing bit error rate, limiting interference to primary users and increasing coverage.
- *Multiple accessing and spectral allocation:* Multiple accessing is inherited in SE-CDMA technique by assigning different codes to different users.
- *NBI immunity:* NBI suppression can be easily done by assigning zero code to the frequency bands in which NBI exists.

To assess the effect of spectrally encoded CDMA in CR, we consider two enabling CR schemes, including overlay and underlay waveforms. We evaluate the performance of the proposed cognitive SE-CDMA scheme in AWGN and fading channels for both primary and secondary users. We also provide simulations to verify analytical results. Further, we compare its performance with the system recently proposed in [3, 4]. The results show that the proposed schemes are efficient.

In this research, it is assumed that the prior knowledge of the spectrum usage is available. In other words, perfect spectral sensing (i.e. probability of detection $P_d = 1$ and probability of false alarm $P_f = 0$) is considered, which allows the secondary users to access the channel. This study essentially focuses on the spectrum shaping in cognitive networks rather than spectrum sensing. Moreover, such assumption is made in the recently introduced studies [3, 4] whose results are compared with our work. Performance analysis for the case of $P_d \neq 1$ and $P_f \neq 0$ is left for future studies.

This paper is organised as follows: Section 2 describes the model of the system. In Sections 3 and 4, we evaluate the performance of the primary and secondary users in AWGN and fading channels, respectively. Section 5 provides analytical and simulation results. Finally, Section 6 summarises the approach and presents future works.

2 System description

In the existing CR systems, secondary users spread their transmit power over the spectrum using techniques such as CDMA or MB-OFDM-based UWB where all these techniques are wideband. In other words, the basic definition of CR has been associated with UWB technology. In 2002, the FCC approval of UWB underlay networks in 3–10 GHz indicated that this frequency range might be opened [19]. In the SE-CDMA context, UWB is a special implementation of the CR waveform. The problem of wideband systems is to handle the dynamic range of spectrum sensing over the bandwidth of several GHz. In recent studies, low complexity decision fusion method has been proposed for cooperative spectrum sensing of wideband CR and has obtained acceptable performance [20]. The same method can be applied when using SE-CDMA in CR networks.

In CR network, spectrum overlay and underlay techniques are employed to enable primary and secondary users to use the spectrum simultaneously. The inherent ability of SE-CDMA to match the transmitted spectrum with the channel spectrum has enabled the SE-CDMA to be implemented in both underlay and overlay modes. In this paper, in order to design the parameters of cognitive system, we study the performance of SE-CDMA technique in AWGN and fading channels for overlay and underlay techniques.

We employ SE-CDMA for secondary users in both underlay and overlay access techniques. We study the performance of primary users in the presence of underlay SE secondary users. Moreover, the performance of SE secondary users in underlay and overlay schemes is analysed, while the primary users are considered as NBIs.

Fig. 1 shows the block diagram of spectrally encoded transmitter system, a typical spectrum for the kth secondary user data in the presence of the primary user and a PN sequence. To encode a pulse, the spectrum of the original data pulse $P(f)$ is multiplied by the spectral code $PN^k(f)$.

The spectrum of $PN^k(f)$ in the baseband consists of $N/2$ distinct chips, each of bandwidth $\Omega_c = (2W/N)$ (W is the bandwidth of the spectrally encoded baseband signal). Noting Fig. 1c, it can be written as follows

$$PN^k(f) = \sum_{i=-N/2}^{N/2-1} c_i^k \operatorname{rect}\left(\frac{f-(i+1/2)\Omega_c}{\Omega_c}\right) \tag{1}$$

where c_i^k is the pseudo-random spectral code of the kth secondary user in the ith chip frequency which takes on the values (+1, −1) for underlay case and the values (+1, 0, −1) for overlay scheme. rect(x) is the rectangle function in the interval (−1/2, 1/2).

As shown in Fig. 1a, the spectrum of data pulse $P(f)$ is multiplied by $PN^k(f)$ in the frequency domain, and then the inverse Fourier transform is applied. Therefore, the transmitted pulse of the kth secondary user will be as

$$q_s^k(t) = F^{-1}\{Q_s^k(f)\} = F^{-1}\{P(f)PN^k(f)\} \tag{2}$$

Considering (1) and for the rectangular

$$P(f) = \begin{cases} 1, & |f| < W \\ 0, & \text{else} \end{cases}$$

we obtain

$$q_s^k(t) = \left[e^{j\pi\Omega_c t}\Omega_c \sin c(\Omega_c t) \sum_{i=-N/2}^{N/2-1} c_i^k e^{i2\pi j\Omega_c t} \right] \tag{3}$$

Note that $c_i^k = c_{-i-1}^k$ to have real $q_s^k(t)$. Because of bandwidth limitation, $q_s^k(t)$ has infinite duration, and therefore it is necessary to apply some time windows. However, it is shown in [4–7] that we can extract more than 90% of energy of the pulse that occurs over duration $2/\Omega_c$.

The transmitted signal of the kth secondary user can be written as

$$s_s^k(t) = \sqrt{E_s^k} \sum_m d_m^k q_s^k(t - mT) \tag{4}$$

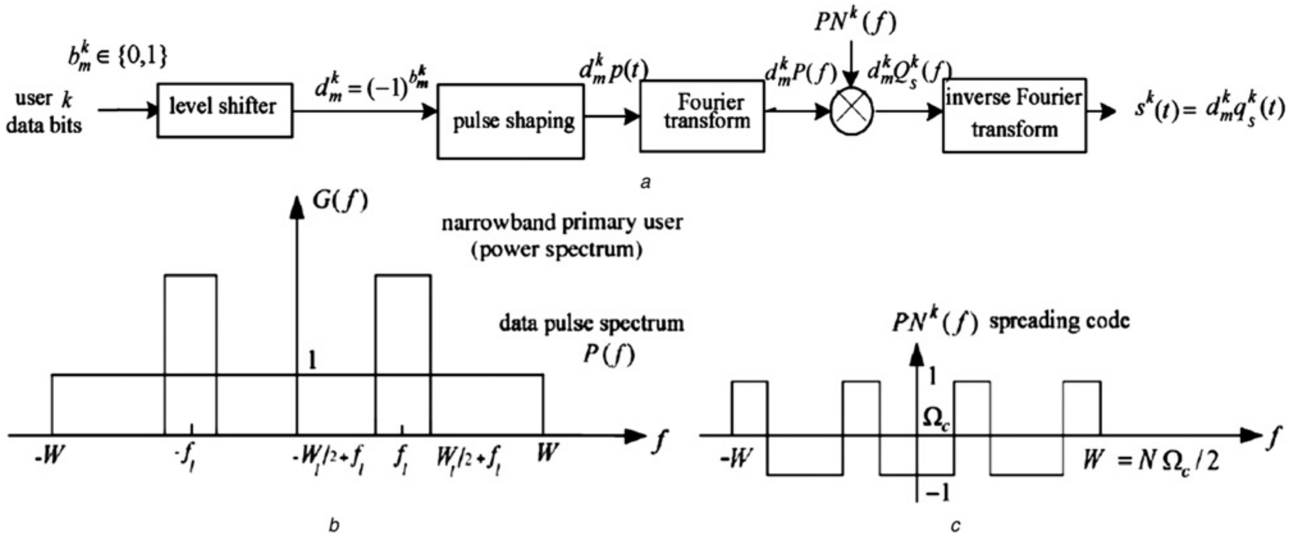

Fig. 1 *Block diagram of spectrally encoded transmitter system, a typical spectrum for the kth secondary user data in the presence of the primary user and a PN sequence*

a Block diagram of SE transmitter for the kth secondary user
b Spectrum of the secondary user in the presence of primary user
c Typical PN sequence

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where $d_m^k = (-1)^{b_m^k}$ and $b_m^k \in \{0,\ 1\}$ is the mth data bit of the kth secondary user, E_s^k is the transmitted energy per bit, and $1/T$ is the data rate.

Fig. 2 shows the block diagram of the receiver for the primary or secondary users, where $r(t)$ is the received signal, $h(t)$ is the impulse response of the channel and $q(t)$ denotes the transmitted pulse.

2.1 System model for underlay approach

In the underlay technique, the signals of the secondary users are spectrally coincident with the primary users. The underlay method for SE cognitive system is illustrated in Fig. 3. As shown, the code sequence takes on the values $\{+1,\ -1\}$ with equal probability, and there is overlap from the spectra of primary users to the secondary user. The primary users are considered as the NBIs for the secondary user.

2.2 System model for overlay approach

Spectrum overlay was first proposed by Mitola [21] using the terminology of spectrum pooling. In this method, radios seek spectrum holes for their communications. To improve spectrum efficiency, overlay systems utilise the unused portions of the spectrum, avoiding the interference to the primary users. In order to achieve this, overlay system needs information about the spectrum allocation of licensed users by regularly performing spectrum measurements. Here, it is assumed that we have information about the frequency bands which are occupied by the primary users.

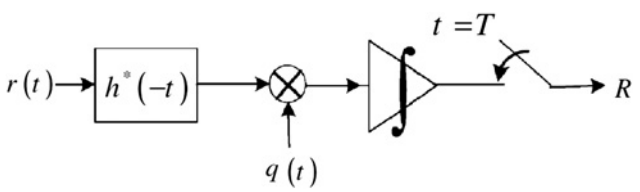

Fig. 2 *Block diagram of the receiver for the desired primary or secondary user*

In our method, for overlay cognitive SE system, we assign code with zero value in the frequency bands of primary users. In this case, the *PN* code sequence takes on the values $\{-1,\ 0,\ +1\}$. Fig. 4 shows the proposed overlay SE signals spectrum.

2.3 Fading channel model

The channel model for the primary user is considered as frequency flat fading. Without loss of generality, it is assumed that the primary user occupies one frequency chip (Ω_c). Hence, the channel model can be written as

$$H_p(f) = \alpha_{f_l}^p \mathrm{rect}\left(\frac{f - f_l}{\Omega_c}\right), \quad \alpha_{f_l}^p = \beta_{f_l}^p \mathrm{e}^{\mathrm{j}\theta_{f_l}^p} \tag{5}$$

where $\alpha_{f_l}^p$ is the complex coefficient of the channel in the frequency chip whose central frequency is f_l, $\beta_{f_l}^p$ is the amplitude having Rayleigh distribution with $E\left(\beta_{f_l}^p\right) = \sigma^2$ and $\theta_{f_l}^p$ is the random phase with uniform distribution. The random variables $\beta_{f_l}^p$ and $\theta_{f_l}^p$ are stochastically independent.

The channel for the secondary user is assumed frequency non-selective in each frequency chip, while it is frequency selective in the bandwidth W, that is, $(\Delta f)_c \ll W$, where $(\Delta f)_c$ is the channel coherence bandwidth [22]. It is also considered that the fading is slow. Therefore, the channel can be modelled as

$$H_s(f) = \sum_{n=-N/2}^{N/2-1} \alpha_n \mathrm{rect}\left(\frac{f - (n + 1/2)\Omega_c}{\Omega_c}\right), \quad \alpha_n = \beta_n \mathrm{e}^{\mathrm{j}\theta_n} \tag{6}$$

where α_n, β_n and θ_n have the same characteristics as mentioned for primary user channel (note that $\beta_n = \beta_{-n-1}$).

Generally, there is a correlation between β_n and $\beta_{n'}$. However, the analysis will be so complex if we consider the correlation and the closed analytical form cannot be obtained. Thus, we assume that β_n and $\beta_{n'}$ are uncorrelated

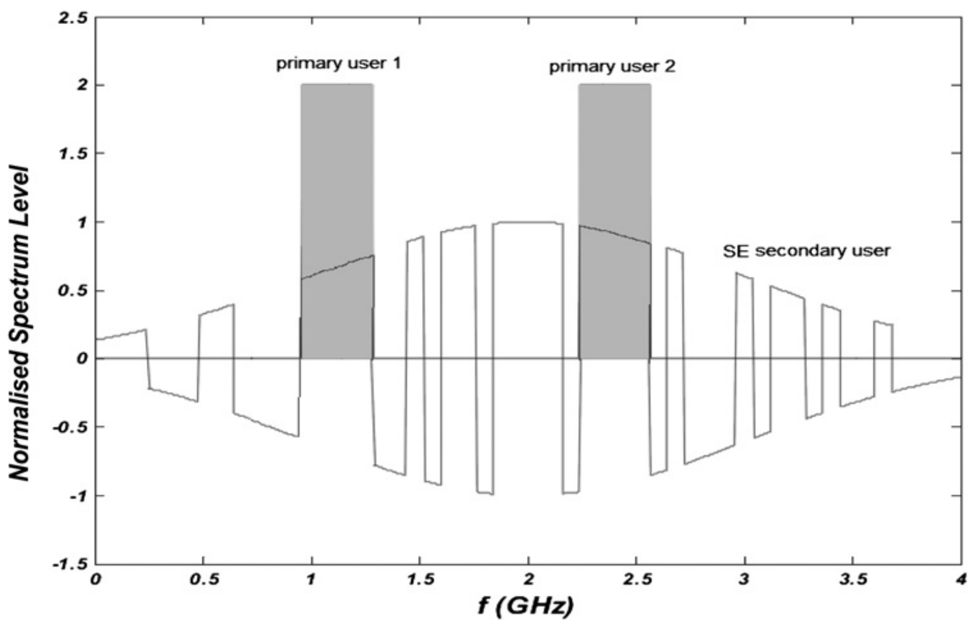

Fig. 3 *Typical underlay cognitive spectrum of spectrally encoded signal*

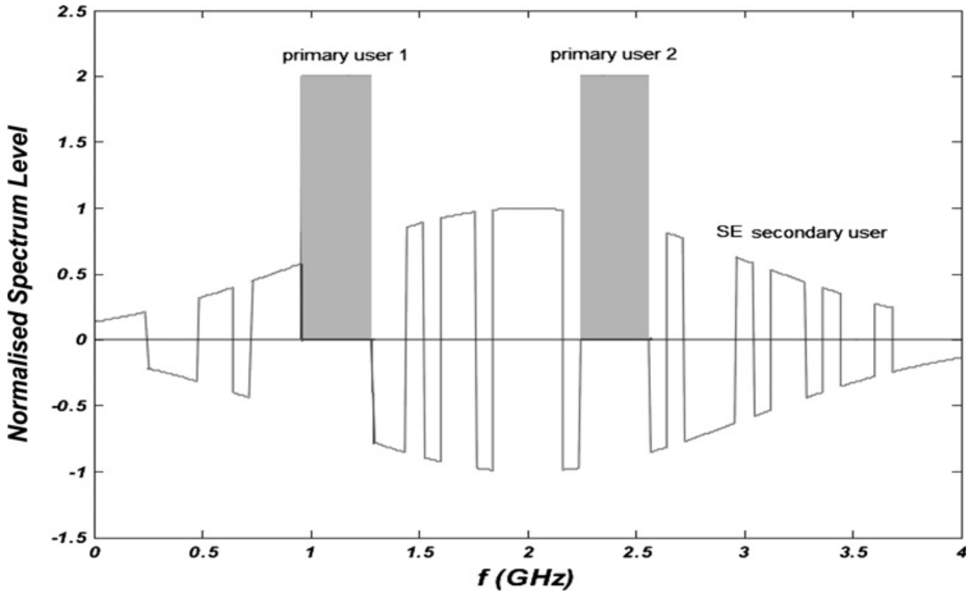

Fig. 4 *Overlay cognitive spectrally encoded signal spectrum*

to obtain closed-form expressions for performance evaluation. It has been shown in [13] that the above channel model well approximates the model of 802.15.4a standard for UWB communications.

3 Performance analysis of primary users

In this section, the effect of the secondary user transmission on the performance of the primary users is investigated. In the overlay technique, since the secondary users' codes are set to zero in the frequency band that the primary user exists, there is no interference from the secondary users to the primary user. That is, in overlay scheme the primary user transmission is simply a non-interference transmission. Therefore, we just consider the primary users performance in the underlay scheme.

As mentioned, we assume that the primary users occupy one frequency chip of the *PN* sequence, that is

$$W_l = \frac{W}{N/2} = \frac{2W}{N} = \Omega_c, \quad l = 1, \ldots, N_p \tag{7}$$

where W_l is the frequency band used by the lth primary user and N_p is the number of primary users in the secondary users frequency band. The received signal at the receiver of the lth primary user for the proposed cognitive system will be as

$$r(t) = s_p^l(t) \otimes h_p(t) + \sum_{k=1}^{N_s} \left(s_s^k(t) \otimes h_s(t) \right) + n(t) \tag{8}$$

where $s_p^l(t)$ is the lth primary user signal, $\otimes$ denotes the convolution operator, N_s is the number of secondary users and $s_s^k(t)$ is the signal of the kth secondary user. $h_p(t)$ and $h_s(t)$ are the impulse responses of the primary and secondary users channels, respectively, and $n(t)$ is a zero-mean Gaussian noise with power spectral density $N_0/2$.

As shown in Fig. 1*b*, the signal of the lth primary user with bandwidth W_l, which is located in the central frequency f_l, in the time and frequency domains is as follows

$$\begin{aligned} s_p^l(t) &= \sqrt{E_p^l} \sum_m d_m^l q_p^l(t - mT); \\ S_p^l(f) &= \sqrt{E_p^l} \left(\sum_m d_m^l \mathrm{e}^{-\mathrm{j}2\pi f m T} \right) Q_p^l(f) \end{aligned} \tag{9}$$

where E_p^l is the transmitted energy per bit of the lth primary user, $d_m^l = (-1)^{b_m^l}$, and $b_m^l \in \{0, 1\}$ is the mth data bit of the lth user; hence d_m^l takes the values $\{+1, -1\}$. $q_p^l(t)$ is the transmitted pulse in the interval $(0, T)$ which has the rectangular spectrum in the frequency domain, that is

$$Q_p^l(f) = \begin{cases} 1, & -W_l/2 + f_l < |f| < W_l/2 + f_l \\ 0, & \text{else} \end{cases} \tag{10}$$

3.1 AWGN channel

The correlator output of the desired (first) primary user in the interval $(0, T)$ noting Fig. 2 and using (7), (9) and (10) is obtained as

$$\begin{aligned} S_p &= \int_0^T s_p^1(t) q_p^1(t)\, \mathrm{d}t = \int_{-\infty}^{\infty} S_p^1(f) Q_p^{1*}(f)\, \mathrm{d}f \\ &= 2 \int_{-W_1/2+f_1}^{W_1/2+f_1} \sqrt{E_p^1} \left(\sum_m d_m^p \mathrm{e}^{-\mathrm{j}2\pi f m T} \right) \left| Q_p^1(f) \right|^2 \mathrm{d}f \\ &= 2\sqrt{E_p^1} d_0^1 \Omega_c \end{aligned} \tag{11}$$

where * denotes complex conjugation. Since we assumed that the primary user occupies one frequency chip of secondary users, to compute the interference, we just consider the secondary user's signal in the occupied frequency chip. As a result, considering the secondary user's signal (4) and the transmitted pulse (2), the correlator output due to the interference from the kth secondary user is computed as follows

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$$I_s^k = \int_0^T s_s^k(t) q_p^1(t)\, dt = \int_{-\infty}^{\infty} S_s^k(f) Q_p^{1*}(f)\, df$$
$$= 2\int_{-W_1/2+f_1}^{W_1/2+f_1} \left(\sqrt{E_s^k}\sum_m d_m^k e^{-j2\pi fmT} Q_s^k(f)\right) Q_p^{1*}(f)\, df$$
$$= 2\int_{-W_1/2+f_1}^{W_1/2+f_1} \left(\sqrt{E_s^k} c_{f_1}^k \sum_m d_m^k e^{-j2\pi fmT}\right) df = 2\sqrt{E_s^k} c_{f_1}^k d_0^k \Omega_c \tag{12}$$

where $c_{f_1}^k$ is the code value of the kth secondary user in the frequency chip that the primary user occupies.

Accordingly, the mean and variance of the kth secondary user's interference are obtained as

$$m_{I_s^k} = 0, \quad \mathrm{Var}(I_s^k) = E\left(\left(I_s^k\right)^2\right)$$
$$= E\left(4E_s^k\left(c_{f_1}^k d_0^k\right)^2 \Omega_c^2\right) = 4E_s^k \Omega_c^2 \tag{13}$$

where $E(\cdot)$ is the expectation operator. Moreover, the variance of noise is calculated as

$$\mathrm{Var}(n) = 2\int_{-W_1/2+f_1}^{W_1/2+f_1} \frac{N_0}{2}\left|Q_p^1(f)\right|^2 df = N_0\Omega_c \tag{14}$$

The bit error rate (BER) expression in AWGN channel can be computed as

$$P_b = Q\left(\sqrt{\mathrm{SINR}}\right) = Q\left(\sqrt{\frac{4E_p^1\Omega_c}{4\Omega_c\sum_{k=1}^{N_s} E_s^k + N_0}}\right) \tag{15}$$

where SINR is the signal-to-interference plus noise ratio and $Q(\cdot)$ is the error function.

3.2 Fading channel

Now, we study the performance of primary users in fading channel. We assume that all secondary users experience the same channel (6) but different from the primary user channel (5). The correlator output of the desired primary user in the interval (0, T) considering Fig. 2, (9) and (10) is computed as

$$S_p = \int_0^T \left(\left(s_p^1(t)\otimes h_p(t)\right)\otimes h_p^*(-t)\right) q_p^1(t)\, dt$$
$$= \int_{-\infty}^{\infty} S_p^1(f) H_p(f) H_p^*(f) Q_p^{1*}(f)\, df$$
$$= 2\int_{-W_1/2+f_1}^{W_1/2+f_1} \left(\sqrt{E_p^1}\sum_m d_m^p e^{-j2\pi fmT}\right)\left|H_p(f)\right|^2 \left|Q_p^1(f)\right|^2 df \tag{16}$$

Noting (5), we obtain

$$\left|H_p(f)\right|^2 = \left(\beta_n^p\right)^2 \mathrm{rect}\left(\frac{f-f_1}{\Omega_c}\right)$$

Thus

$$S_p = 2\int_{-W_1/2+f_1}^{W_1/2+f_1} \left(\sqrt{E_p^1}\sum_m d_m^1 e^{-j2\pi fmT}\right)\left(\beta_{f_1}^p\right)^2 \mathrm{rect}\left(\frac{f-f_1}{\Omega_c}\right) df$$
$$= 2\sqrt{E_p^1} d_0^1 \Omega_c \left(\beta_{f_1}^p\right)^2 \tag{17}$$

The interference term from the kth secondary user noting (4), (5), (6) and (10) is calculated as

$$I_s^k = \int_0^T \left(\left(s_s^k(t)\otimes h_s(t)\right)\otimes h_p^*(-t)\right) q_p^1(t)\, dt$$
$$= \int_{-\infty}^{\infty} S_s^k(f) H_s(f) H_p^*(f) Q_p^{1*}(f)\, df$$
$$= 2\int_{-\infty}^{\infty} \left(\sqrt{E_s^k}\sum_m d_m^k e^{-j2\pi fmT}\right) c_{f_1}^k \alpha_{f_1} \mathrm{rect}\left(\frac{f-f_1}{\Omega_c}\right)\alpha_{f_1}^{p*}\mathrm{rect}\left(\frac{f-f_1}{\Omega_c}\right) df \tag{18}$$
$$= 2\sqrt{E_s^k} d_0^k c_{f_1}^k \Omega_c \beta_{f_1}\beta_{f_1}^p e^{\theta_{f_1}-\theta_{f_1}^p}$$

The conditional variance of interference conditioned on $\beta_{f_1}^p$ is obtained as follows

$$\mathrm{Var}\left(I_s^k\middle|\beta_{f_1}^p\right) = E\left(\left(I_s^k\right)^2\middle|\beta_{f_1}^p\right)$$
$$= E\left(4E_s^k\Omega_c^2\left(\beta_{f_1}\right)^2\left(\beta_{f_1}^p\right)^2\middle|\beta_{f_1}^p\right) = 4E_s^k\Omega_c^2\sigma^2\left(\beta_{f_1}^p\right)^2 \tag{19}$$

The variance due to noise is also computed as

$$\mathrm{Var}\left(n\middle|\beta_{f_1}^p\right) = \frac{N_0}{2}\times 2\int_{-W_1/2+f_1}^{W_1/2+f_1} \left|H_p(f)\right|^2\left|Q_p^1(f)\right|^2 df$$
$$= N_0\Omega_c\left(\beta_{f_1}^p\right)^2 \tag{20}$$

By defining

$$\gamma_b = \mathrm{SINR} = \frac{4E_p^1\Omega_c\left(\beta_{f_1}^p\right)^2}{4\Omega_c\sigma^2\sum_{k=1}^{N_s} E_s^k + N_0}$$

the BER expression in fading channel for the primary user can be obtained as follows [22]

$$P_b = \int Q\left(\sqrt{\gamma_b}\right) p(\gamma_b)\, d\gamma_b = \frac{1}{2}\left(1-\sqrt{\frac{\bar{\gamma}_b}{1+\bar{\gamma}_b}}\right) \tag{21}$$

where $\bar{\gamma}_b$ is the average SINR computed as

$$\bar{\gamma}_b = E[\gamma_b] = \frac{4E_p^1\Omega_c\sigma^2}{4\Omega_c\sigma^2\sum_{k=1}^{N_s} E_s^k + N_0} \tag{22}$$

4 Performance evaluation of secondary users

In this section, we study the performance of secondary users in the presence of primary users. The secondary users transmit their signals using spectrally encoded CDMA with binary phase shift keying (BPSK) modulation. We evaluate the performance of secondary user in two cases. In the first case, we assume that the primary users are narrowband Gaussian processes and in the second scheme, we obtain the exact analysis of NBI due to the primary users signals. We consider downlink transmission where the secondary users are synchronous and experience the same fading channel.

4.1 Performance evaluation for underlay approach

As mentioned, we use two level codes for SE secondary users in the underlay scheme. Thus, there is interference from primary users to the secondary users which must be considered. In the following, we study the performance of underlay secondary user in AWGN and fading channels for both Gaussian assumption and exact analysis of primary users' interferences.

4.1.1 AWGN channel: The received signal at the receiver for the proposed cognitive system will be as follows

$$r(t) = \sum_{k=1}^{N_s} s_s^k(t) + \sum_{l=1}^{N_p} I_p^l + n(t) \tag{23}$$

where I_p^l is the NBI due to the lth primary user.

Fig. 2 shows the block diagram of the receiver in the proposed system. Assuming that the receiver has the information of the *PN* code of desired (first) user, the correlator correlates the received signal with $q_s^1(t)$, which is defined in (3).

The correlator output for the desired secondary user in the interval $(0, T)$ is

$$\begin{aligned} S &= \int_0^T s_s^1(t) q_s^1(t)\,\mathrm{d}t = \int_{-\infty}^{\infty} S_s^1(f) Q_s^{1*}(f)\,\mathrm{d}f \\ &= \int_{-W}^{W} \sqrt{E_s^1}\left(\sum_m d_m^1 \mathrm{e}^{-\mathrm{j}2\pi f m T}\right)\left|Q_s^1(f)\right|^2 \mathrm{d}f \end{aligned} \tag{24}$$

Noting (2), it can be easily proved that $\left|Q_s^1(f)\right|^2 = \left|P(f)\right|^2$. Thus, we have

$$\begin{aligned} S &= \int_{-W}^{W}\left(\sqrt{E_s^1}\sum_m d_m^1 \mathrm{e}^{-\mathrm{j}2\pi f m T}\right)\left|P(f)\right|^2 \mathrm{d}f \\ &= \sqrt{E_s^1} N \Omega_c d_0^1 \end{aligned} \tag{25}$$

where we assumed that $P(f)$ has the rectangular pulse shape in the interval $(-W, W)$ as shown in Fig. 1*b*.

The correlator output due to the interference from the kth secondary user noting (2) and (4) is obtained as

$$\begin{aligned} I_{\mathrm{MAI}}^k &= \int_{-\infty}^{\infty} S_s^k(f) Q_s^{1*}(f)\,\mathrm{d}f = \int_{-W}^{W}\left(\sqrt{E_s^k}\sum_m d_m^k \mathrm{e}^{-\mathrm{j}2\pi f m T}\right) \\ &\quad \left(\sum_{i=-N/2}^{N/2-1} c_i^k c_i^1 \,\mathrm{rect}\left(\frac{f-(i+1/2)\Omega_c}{\Omega_c}\right)\right)\mathrm{d}f \\ &= \sqrt{E_s^k} d_0^k \Omega_c \sum_{i=-N/2}^{N/2-1} c_i^k c_i^1 \end{aligned} \tag{26}$$

The mean and variance of the interference from the kth secondary user are found as

$$m_{\mathrm{MAI}} = 0, \quad \mathrm{Var}\left(I_{\mathrm{MAI}}^k\right) = E_s^k N \Omega_c^2 \tag{27}$$

where we assumed that the codes of secondary users are independent. Noise variance can be easily computed as

$$\mathrm{Var}(n) = \frac{N_0}{2}\int_{-W}^{W}\left|Q_s^1(f)\right|^2 \mathrm{d}f = \frac{N_0}{2} 2W = \frac{N_0}{2} N \Omega_c \tag{28}$$

Gaussian assumption for narrowband primary users' interferences: It has been shown that narrowband OFDM signals can be modelled as Gaussian interference upon the UWB systems [23]. In other work, it is proven that if a general tone is considered as a synchronous interferer, the interference term is zero mean Gaussian process. In addition, WiMax is a narrowband interferer for UWB systems. In the recent study [24], the authors examined WiMax interference as Gaussian. As a result, we can assume the interference of primary users zero-mean Gaussian processes.

To study the effect of interference from the primary users, noting Fig. 1*b*, we consider the power spectrum of the lth primary user as

$$G^l(f) = \begin{cases} J_p^l/2, & -W_l/2 + f_l < |f| < W_l/2 + f_l \\ 0, & \text{else} \end{cases} \tag{29}$$

where W_l is the lth primary user's bandwidth and f_l is the central frequency of the lth primary user. The variance of the interference due to the lth primary user $\left(I_{\mathrm{NBI}}^l\right)$ is computed as

$$\begin{aligned} \mathrm{Var}\left(I_{\mathrm{NBI}}^l\right) &= 2 \times \frac{J_p^l}{2}\int_{-W_l/2+f_l}^{W_l/2+f_l}\left|Q_s^1(f)\right|^2 \mathrm{d}f \\ &= 2 \times \frac{J_p^l}{2}\int_{-W_l/2+f_l}^{W_l/2+f_l}\left|P(f)\right|^2 \mathrm{d}f = J_p^l \Omega_c \end{aligned} \tag{30}$$

The decision variable (output) at the receiver of the desired secondary user (first user) is

$$\begin{aligned} R &= \sqrt{E_s^1} N \Omega_c d_0^1 + \sum_{k=2}^{N_s}\left(\sqrt{E_s^k}\Omega_c d_0^k \sum_{i=-N/2}^{N/2-1} c_i^k c_i^1\right) \\ &\quad + \sum_{l=1}^{N_p} I_{\mathrm{NBI}}^l + n \end{aligned} \tag{31}$$

where n is the output Gaussian noise with zero mean. The SINR can be written as follows

$$\begin{aligned} \mathrm{SINR} &= \frac{E^2(s)}{\mathrm{var}\left(I_{\mathrm{MAI}}\right) + \mathrm{var}\left(I_{\mathrm{NBI}}\right) + \mathrm{var}(n)} \\ &= \frac{E_s^1 N \Omega_c}{\sum_{k=2}^{N_s} E_s^k \Omega_c + (1/N)\sum_{l=1}^{N_p} J_p^l + \left((N_0)/2\right)} \end{aligned} \tag{32}$$

Thus, the BER can be obtained as

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$$P_b = Q\left(\sqrt{\text{SINR}}\right) = Q\left(\sqrt{\frac{E_s^1 N\Omega_c}{\sum_{k=2}^{N_s} E_s^k \Omega_c + (1/N)\sum_{l=1}^{N_p} J_p^l + \left((N_0)/2\right)}}\right) \tag{33}$$

Exact analysis of narrowband primary users' interferences: Now, we calculate the exact value of NBI from primary users and then we will obtain the BER expression.

The correlator output due to the interference signal of the lth primary user is

$$I_{\text{NBI}}^l = \int_{-\infty}^{\infty} S_p^l(f) Q_s^{1*}(f)\, df = 2\int_{-W_l/2+f_l}^{W_l/2+f_l} \left(\sqrt{E_p^l}\sum_m d_m^l e^{-j2\pi fmT}\right) Q_p^l(f) Q_s^{1*}(f)\, df = 2\sqrt{E_p^l} d_0^l \Omega_c c_{f_l}^1 \tag{34}$$

The mean and variance of the NBI are computed as

$$m_{\text{MAI}} = 0, \quad \text{Var}\left(I_{\text{NBI}}^l\right) = 4E_p^l \Omega_c^2 \tag{35}$$

Consequently, the BER can be written as

$$P_b = Q\left(\sqrt{\text{SINR}}\right) = Q\left(\sqrt{\frac{E_s^1 N\Omega_c}{\sum_{k=2}^{N_s} E_s^k \Omega_c + (1/N)\sum_{l=1}^{N_p} 4E_p^l \Omega_c + \left((N_0)/2\right)}}\right) \tag{36}$$

4.1.2 Fading channel: Now, we analyse the performance of underlay spectrally encoded secondary user in fading channel. The received signal is

$$r(t) = \sum_{k=1}^{N_s}\left(\sqrt{E_s^k}\sum_m d_m^k q_s^k(t - mT)\right) \otimes h_s(t) + \sum_{l=1}^{N_p} I_p^l + n(t) \tag{37}$$

The correlator output for the desired secondary user in the interval $(0, T)$ can be obtained as

$$S = \int_0^T \left(\left(s_s^1(t) \otimes h_s(t)\right) \otimes h_s^*(-t)\right) q_s^1(t)\, dt = \int_{-\infty}^{\infty} S_s^1(f) H_s(f) H_s^*(f) Q_s^{1*}(f)\, df = \int_{-W}^{W}\left(\sqrt{E_s^1}\sum_m d_m^1 e^{-j2\pi fmT}\right)\left|H_s(f)\right|^2 \left|Q_s^1(f)\right|^2 df \tag{38}$$

From (6), we note that

$$\left|H_s(f)\right|^2 = \sum_{n=-N/2}^{N/2-1} \beta_n^2 \text{rect}\left(\frac{f - (n + 1/2)\Omega_c}{\Omega_c}\right)$$

Thus, we have

$$S = \int_{-W}^{W} \left|P(f)\right|^2 \left(\sum_{n=-N/2}^{N/2-1} \beta_n^2 \text{rect}\left(\frac{f - \left(n + 1/2\right)\Omega_c}{\Omega_c}\right)\right)\left(\sqrt{E_s^1}\sum_m d_m^1 e^{-j2\pi fmT}\right) df = 2\sqrt{E_s^1} d_0^1 \Omega_c \sum_{n=0}^{N/2-1} \beta_n^2 \tag{39}$$

The correlator output due to the interference from the kth secondary user is found as

$$I_{\text{MAI}}^k = \int_{-\infty}^{\infty} S_s^k(f) H_s(f) H_s^*(f) Q_s^{1*}(f)\, df = 2\sqrt{E_s^k} d_0^k \Omega_c \sum_{n=0}^{N/2-1} \beta_n^2 c_n^k c_n^1 \tag{40}$$

The conditional variances of interference from the secondary users and noise are computed as

$$\text{Var}\left(I_{\text{MAI}}^k \middle| \beta_n\right) = E\left[\left(I_{\text{MAI}}^k\right)^2 \middle| \beta_n\right] = 4E_s^k \Omega_c^2 \sum_{n=0}^{N/2-1} \beta_n^4 \tag{41}$$

$$\text{Var}\left(n \middle| \beta_n\right) = \frac{N_0}{2}\int_{-W}^{W} \left|H(f)\right|^2 \left|Q_s^1(f)\right|^2 df = N_0 \Omega_c \sum_{n=0}^{N/2-1} \beta_n^2 \tag{42}$$

Gaussian assumption for narrowband primary users' interferences: The variance of interference owing to the lth primary user is computed as (see (43))

In slow fading channel, the BER cannot be computed directly like as AWGN channel, because the SINR is a function of β_n^2 and the closed form of error probability cannot be computed. Accordingly, we use the Beaulieu series, which is an infinite series for the computation of the cumulative distribution function (CDF) of a random variable using the samples of its moment-generating function [25]. From this series, we have

$$P_b = P\left(R = S + \text{MAI} + \text{NBI} + n < 0 \middle| d_0^1 = 1\right) = \frac{1}{2} - \sum_{\substack{m=1 \\ m \in N_{\text{odd}}}} \frac{2\text{Im}\left\{\varphi_R\left(jm\omega_o\right) \middle| d_0^1 = 1\right\}}{m\pi} \tag{44}$$

where N_{odd} is the set of odd natural numbers, $\phi_R(s)$ is the moment-generating function of R and ω_o is the sampling

$$\text{Var}\left(I_{\text{NBI}}^l \middle| \beta_{f_l}\right) = 2 \times \frac{J_p^l}{2}\int_{-W_l/2+f_l}^{W_l/2+f_l} \left|H_s(f)\right|^2 \left|Q_s^1(f)\right|^2 df = 2 \times \frac{J_p^l}{2}\int_{-W_l/2+f_l}^{W_l/2+f_l} \left(\sum_{n=0}^{N/2-1} \beta_n^2 \text{rect}\left(\frac{f - \left(n + 1/2\right)\Omega_c}{\Omega_c}\right)\right) df = J_p^l \Omega_c \beta_{f_l}^2 \tag{43}$$

rate. $\phi_R(s)$ can be computed as follows

$$\varphi_{(R=S+\mathrm{MAI}+\mathrm{NBI}+n)\left|d_0^1=1\right.}(s)=\int \varphi_{R\left|\beta_{\{n\}}\right.}(s)p\left(\beta_{\{n\}}\right)\,\mathrm{d}\beta_{\{n\}} \tag{45}$$

To obtain the above integral, at first R and then $\varphi_{R\left|\beta_{\{n\}}\right.}(s)$ must be calculated. We note that

$$\begin{aligned} R &= 2\sqrt{E_\mathrm{s}^1}d_0^1\Omega_\mathrm{c}\sum_{n=0}^{N/2-1}\beta_n^2 \\ &+\sum_{k=2}^{N_\mathrm{s}}\left(2\sqrt{E_\mathrm{s}^k}d_0^k\Omega_\mathrm{c}\sum_{n=0}^{N/2-1}\beta_n^2c_n^kc_n^1\right)+\sum_{l=1}^{N_p}I_{\mathrm{NBI}}^l+n \end{aligned} \tag{46}$$

Since n and I_{NBI}^l are Gaussian processes, it can be verified that $R|_{\beta_{\{n\}},\,d_0^k,\,c_{\{n\}}^k,\,c_{\{n\}}^1}=\text{constant}+n$. Thus, we have

$$\begin{aligned} R|_{\beta_{\{n\}},\,d_0^k,\,c_{\{n\}}^k,\,c_{\{n\}}^1} &\sim N\left(2\sqrt{E_\mathrm{s}^1}d_0^1\Omega_\mathrm{c}\sum_{n=0}^{N/2-1}\beta_n^2+\sum_{k=2}^{N_\mathrm{s}}2\sqrt{E_\mathrm{s}^k}d_0^k\Omega_\mathrm{c}\right. \\ &\left.\sum_{n=0}^{N/2-1}\beta_n^2c_n^kc_n^1,\ \sum_{l=1}^{N_\mathrm{p}}J_\mathrm{p}^l\Omega_\mathrm{c}\beta_{f_l}^2+\sigma_n^2\right) \end{aligned} \tag{47}$$

where $N(m,\,\sigma^2)$ denotes Gaussian distribution with mean m and variance σ^2. The variables d_0^k, c_n^k and c_n^1 take on the values +1 or −1 with equal probability. Here, we assume that the energy of secondary users is the same (E_s) and all primary users have the same power spectrum as $J_\mathrm{p}/2$. Thus, the moment-generating function conditioned on the fading coefficients can be written as (see (48))

By replacing (48) in (45), the moment-generating function is derived as (see (49))

Noting that β_n has Rayleigh distribution, using a change of variable $\lambda_n=\beta_n^2$, it can be verified that

$$P_{\lambda_n}\left(\lambda_n\right)=\frac{1}{\sigma^2}\exp\left(-\frac{\lambda_n}{\sigma^2}\right)\quad E\left(\lambda_n=\beta_n^2\right)=\sigma^2 \tag{50}$$

Considering the binomial expansion of the first term of the integral in (49), the moment-generating function is obtained as follows (see (51))

Therefore, we have (see (52))

By substituting (52) in (44), the probability of error can be obtained.

Exact analysis of narrowband primary users' interferences: Here, we consider that all primary and secondary users experience the same channel. The interference signal due to the lth primary user is computed

$$\begin{aligned} \varphi_{R|\beta\{n\}}(s) &= \prod_{n=0}^{N/2-1}\left[\left(1/2\exp\left\{2s\sqrt{E_\mathrm{s}}\Omega_\mathrm{c}\beta_n^2\right\}+1/2\exp\left\{-2s\sqrt{E_\mathrm{s}}\Omega_\mathrm{c}\beta_n^2\right\}\right)^{N_s-1}\exp\left\{\left(s^2\left(N_0/2\right)+2s\sqrt{E_\mathrm{s}}\right)\Omega_\mathrm{c}\beta_n^2\right\}\right] \\ &\prod_{l=1}^{N_\mathrm{p}}\exp\left\{s^2\left(J_\mathrm{p}/2\right)\Omega_\mathrm{c}\beta_{f_l}^2\right\} \end{aligned} \tag{48}$$

$$\begin{aligned} \varphi_{R|d_0^1=1}(s) &= \int \varphi_{R|\beta_{\{n\}}}(s)p(\beta_{\{n\}})\,\mathrm{d}\beta_{\{n\}} \\ &= \int_0^\infty\cdots\int_0^\infty\prod_{n=0}^{N/2-1}[1/2\exp\{2s\sqrt{E_\mathrm{s}}\Omega_\mathrm{c}\beta_n^2\}+1/2\exp\{-2s\sqrt{E_\mathrm{s}}\Omega_\mathrm{c}\beta_n^2\})^{N_s-1}\exp\{(s^2(N_0/2)+2s\sqrt{E_\mathrm{s}})\Omega_\mathrm{c}\beta_n^2\}] \\ &\prod_{l=1}^{N_p}\exp\{s^2(J_\mathrm{p}/2)\Omega_\mathrm{c}\beta_{f_l}^2\}p(\beta_0,\ \ldots,\ \beta_n,\ \ldots,\ \beta_{(N/2)-1})d\beta_0,\ \ldots,\ d\beta_n,\ \ldots,\ d\beta_{(N/2)-1} \end{aligned} \tag{49}$$

$$\begin{aligned} \varphi_{R\left|d_0^1=1\right.}(s) &= \int \varphi_{R|\beta_{\{n\}}}(s)p\left(\beta_{\{n\}}\right)d\beta_{\{n\}} \\ &= \int_0^\infty\ldots\int_0^\infty\left(\prod_{n=0}^{N/2-1}\left(\frac{1}{2}\right)^{N_\mathrm{s}-1}\sum_{k=0}^{N_\mathrm{s}-1}\binom{N_\mathrm{s}-1}{k}\exp\left\{-2s\left(N_\mathrm{s}-1-2k\right)\sqrt{E_\mathrm{s}}\Omega_\mathrm{c}\lambda_n\right\}\right)\prod_{l=1}^{N_\mathrm{p}}\exp\left\{s^2\left(J_p/2\right)\Omega_\mathrm{c}\lambda_{f_l}\right\} \\ &\left(\prod_{n=0}^{N/2-1}\exp\left\{\left(\frac{s^2}{2}N_0+2s\sqrt{E_\mathrm{s}}\right)\Omega_\mathrm{c}\lambda_n\right\}\right)\left(\prod_{n=0}^{N/2-1}\frac{1}{\sigma^2}\exp\left(-\frac{\lambda_n}{\sigma^2}\right)\right)d\lambda_0,\ \ldots,\ d\lambda_n,\ \ldots,\ d\lambda_{(N/2)-1} \end{aligned} \tag{51}$$

$$\begin{aligned} \varphi_{R\left|d_0^1=1\right.}(s) &= \left[\left(\frac{1}{2}\right)^{N_\mathrm{s}-1}\sum_{k=0}^{N_\mathrm{s}-1}\binom{N_\mathrm{s}-1}{k}\times\frac{1}{2s\left(N_\mathrm{s}-2-2k\right)\sqrt{E_\mathrm{s}}\Omega_\mathrm{c}\sigma^2-\frac{1}{2}s^2\Omega_\mathrm{c}\sigma^2\left(N_0+J_\mathrm{p}\right)+1}\right]^{N_\mathrm{p}} \\ &\left[\left(\frac{1}{2}\right)^{N_\mathrm{s}-1}\sum_{k=0}^{N_\mathrm{s}-1}\binom{N_s-1}{k}\times\frac{1}{2s\left(N_\mathrm{s}-2-2k\right)\sqrt{E_\mathrm{s}}\Omega_\mathrm{c}\sigma^2-s^2\Omega_\mathrm{c}\sigma^2\left(\left(N_0\right)/2\right)+1}\right]^{(N/2)-N_p} \end{aligned} \tag{52}$$

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as (see (53))

The mean and conditional variance of interference is found as follows

$$m_{\mathrm{MAI}} = 0, \quad \mathrm{Var}\left(I^l_{\mathrm{NBI}}\middle|\beta_{f_l}\right) = 4E^l_{\mathrm{p}}\Omega^2_{\mathrm{c}}\beta^4_{f_l} \tag{54}$$

Considering the decision variable in (46), we have

$$\begin{aligned} R|_{\beta_{\{n\}},\, d^k_0,\, c^k_{\{n\}},\, c^1_{\{n\}}} &\sim N\left(2\sqrt{E^1_{\mathrm{s}}}d^1_0\Omega_{\mathrm{c}}\sum_{n=0}^{N/2-1}\beta^2_n + 2\sum_{k=2}^{N_s}\left(\sqrt{E^k_{\mathrm{s}}}d^k_0\Omega_{\mathrm{c}}\sum_{n=0}^{N/2-1}\beta^2_n c^k_n c^1_n\right)\right. \\ &\left. + 2\sum_{l=1}^{N_{\mathrm{p}}}\left(\sqrt{E^l_{\mathrm{p}}}d^l_0\Omega_{\mathrm{c}}c^1_{f_l}\beta^2_{f_l}\right), \quad \sigma^2_n\right) \end{aligned} \tag{55}$$

The moment-generating function conditioned on the fading coefficients can be written as (see (56))

By replacing (56) in (45), the moment-generating function is derived as follows (see (57))

Considering the binomial expansion in (57), the moment-generating function is computed as (see (58))

Thereby, after some manipulations, it is easily obtained that (see (59))

Consequently, the probability of error can be calculated from (44).

4.2 Performance evaluation for overlay approach

As mentioned before, in overlay scheme we assign zero value codes to the frequency chips, in which the primary users exist. Since we assumed that each primary user occupies one frequency chip, the number of zero value codes is equal to the number of primary users (N_{p}). Therefore, the number of frequency chips in *PN* sequence, which have the values +1 or −1 are $N/2-N_{\mathrm{p}}$ and the interference from primary users will be cancelled. Thus, the computations for overlay SE

$$\begin{aligned} I^l_{\mathrm{NBI}} &= \int_{-\infty}^{\infty} S^l_{\mathrm{p}}(f)H_{\mathrm{s}}(f)H^*_{\mathrm{s}}(f)Q^{1*}_{\mathrm{s}}(f)\,\mathrm{d}f = 2\int_{-W_l/2+f_l}^{W_l/2+f_l}\left(\sqrt{E^l_{\mathrm{p}}}\sum_m d^l_m \mathrm{e}^{-\mathrm{j}2\pi fmT}\right)Q^l_{\mathrm{p}}(f)H_{\mathrm{s}}(f)H^*_{\mathrm{s}}(f)Q^{1*}_{\mathrm{s}}(f)\,\mathrm{d}f \\ &= 2\sqrt{E^l_{\mathrm{p}}}d^l_0\Omega_{\mathrm{c}}c^1_{f_l}\beta^2_n = 2\sqrt{E^l_{\mathrm{p}}}d^l_0\Omega_{\mathrm{c}}c^1_{f_l}\beta^2_{f_l} \end{aligned} \tag{53}$$

$$\begin{aligned} \varphi_{R|\beta\{n\}}(s) = &\prod_{n=0}^{N/2-1}\left[\left(1/2\exp\left\{2s\sqrt{E_{\mathrm{s}}}\Omega_{\mathrm{c}}\beta^2_n\right\} + 1/2\exp\left\{-2s\sqrt{E_{\mathrm{s}}}\Omega_{\mathrm{c}}\beta^2_n\right\}\right)^{N_s-1}\exp\left\{\left(s^2\left(N_0/2\right) + 2s\sqrt{E_{\mathrm{s}}}\right)\Omega_{\mathrm{c}}\beta^2_n\right\}\right] \\ &\times\left(1/2\exp\left\{2s\sqrt{E_{\mathrm{p}}}\Omega_{\mathrm{c}}\beta^2_{f_l}\right\} + 1/2\exp\left\{-2s\sqrt{E_{\mathrm{p}}}\Omega_{\mathrm{c}}\beta^2_{f_l}\right\}\right)^{N_p} \end{aligned} \tag{56}$$

$$\begin{aligned} \varphi_{R|_{d^1_0=1}}(s) &= \int\varphi_{R|\beta_{\{n\}}}(s)p\left(\beta_{\{n\}}\right)d\beta_{\{n\}} = \int_0^{\infty}\cdots\int_0^{\infty}\prod_{n=0}^{N/2-1}\left[\left(1/2\exp\left\{2s\sqrt{E_{\mathrm{s}}}\Omega_{\mathrm{c}}\beta^2_n\right\}+1/2\exp\left\{-2s\sqrt{E_{\mathrm{s}}}\Omega_{\mathrm{c}}\beta^2_n\right\}\right)^{N_s-1}\right. \\ &\left.\times\exp\left\{\left(s^2\left(N_0/2\right) + 2s\sqrt{E_{\mathrm{s}}}\right)\Omega_{\mathrm{c}}\beta^2_n\right\}\right]\left(1/2\exp\left\{2s\sqrt{E_{\mathrm{p}}}\Omega_{\mathrm{c}}\beta^2_{f_l}\right\} + 1/2\exp\left\{-2s\sqrt{E_{\mathrm{p}}}\Omega_{\mathrm{c}}\beta^2_{f_l}\right\}\right)^{N_{\mathrm{p}}} \\ &p\left(\beta_0,\ \ldots,\ \beta_n,\ \ldots,\ \beta_{(N/2)-1}\right)d\beta_0,\ \ldots,\ d\beta_n,\ \ldots,\ d\beta_{(N/2)-1} \end{aligned} \tag{57}$$

$$\begin{aligned} \varphi_{R|_{d^1_0=1}}(s) &= \int\varphi_{R|\beta_{\{n\}}}(s)p\left(\beta_{\{n\}}\right)d\beta_{\{n\}} = \int_0^{\infty}\ldots\int_0^{\infty}\left(\prod_{n=0}^{N/2-1}\left(\frac{1}{2}\right)^{N_{\mathrm{s}}-1}\sum_{k=0}^{N_{\mathrm{s}}-1}\binom{N_{\mathrm{s}}-1}{k}\exp\left\{-2s\left(N_{\mathrm{s}}-1-2k\right)\sqrt{E_{\mathrm{s}}}\Omega_{\mathrm{c}}\lambda_n\right\}\right) \\ &\times\left(\left(\frac{1}{2}\right)^{N_{\mathrm{p}}}\sum_{l=0}^{N_{\mathrm{p}}}\binom{N_{\mathrm{p}}}{l}\exp\left\{-2s\left(N_{\mathrm{p}}-2l\right)\sqrt{E_{\mathrm{p}}}\Omega_{\mathrm{c}}\lambda_{f_l}\right\}\right)\left(\prod_{n=0}^{N/2-1}\exp\left\{\left(\frac{s^2}{2}N_0 + 2s\sqrt{E_{\mathrm{s}}}\right)\Omega_{\mathrm{c}}\lambda_n\right\}\right) \\ &\times\left(\prod_{n=0}^{N/2-1}\frac{1}{\sigma^2}\exp\left(-\frac{\lambda_n}{\sigma^2}\right)\right)d\lambda_0,\ \ldots,\ d\lambda_n,\ \ldots,\ d\lambda_{(N/2)-1} \end{aligned} \tag{58}$$

$$\begin{aligned} \varphi_{R|_{d^1_0=1}}(s) &= \left(\frac{1}{2}\right)^{N_p}\left(\frac{1}{2}\right)^{(N/2)\left(N_s-1\right)}\left[\sum_{k=0}^{N_{\mathrm{s}}-1}\binom{N_{\mathrm{s}}-1}{k}\times\frac{1}{2s\left(N_{\mathrm{s}}-2-2k\right)\sqrt{E_{\mathrm{s}}}\Omega_{\mathrm{c}}\sigma^2 - s^2\Omega_{\mathrm{c}}\sigma^2\left(\left(N_0\right)/2\right)+1}\right]^{(N/2)-1} \\ &\times\sum_{l=0}^{N_{\mathrm{p}}}\left\{\binom{N_{\mathrm{p}}}{l}\sum_{k=0}^{N_s-1}\binom{N_s-1}{k}\times\frac{1}{2s\left(N_{\mathrm{s}}-2-2k\right)\sqrt{E_{\mathrm{s}}}\Omega_{\mathrm{c}}\sigma^2 + 2s\left(N_{\mathrm{p}}-2l\right)\sqrt{E_{\mathrm{p}}}\Omega_{\mathrm{c}}\sigma^2 - (1/2)s^2\Omega_{\mathrm{c}}\sigma^2N_0 + 1}\right\} \end{aligned} \tag{59}$$

system are similar to the underlay scheme with a little difference. We must replace the number of frequency chips $N/2$ with $N/2 - N_p$. In addition, the primary users' interferences do not exist. Consequently, the expression for BER analysis in AWGN channel noting (33) will be as

$$P_b = Q\left(\sqrt{\text{SINR}}\right) = Q\left(\sqrt{\frac{E_s^1\left(N - 2N_p\right)\Omega_c}{\sum_{k=2}^{N_s} E_s^k\Omega_c + \left((N_0)/2\right)\left(N/\left(N - 2N_p\right)\right)}}\right) \tag{60}$$

Moreover, in fading channel the moment-generating function will be as (see (61))
Then, the error probability can be obtained from (44).

5 Numerical results

In this section, we present analytical expressions and simulation results in different cases to evaluate the performance of the proposed method. Analytic and simulated P_b against SNR and the number of secondary and primary users are used as the performance metrics to validate the underlay and overlay SE methods. To this end, we consider two channels, AWGN and fading, for underlay and overlay techniques. Finally, we compare the results of cognitive SE-CDMA with the SD-SMSE method proposed in [3, 4].

Noting (25) and (28), we define the SNR of underlay and overlay schemes for performance analysis of secondary users in AWGN channel as

$$\text{SNR}_{\text{underlay}} = \frac{E_s N \Omega_c}{N_0/2}, \quad \text{SNR}_{\text{overlay}} = \frac{E_s\left(N - 2N_p\right)\Omega_c}{\left(N/\left(N - 2N_p\right)\right)N_0/2} \tag{62}$$

Also, the average SNR in fading channel is

$$\text{SNR}_{\text{underlay}} = \frac{E_s N \Omega_c \sigma^2}{N_0/2}, \quad \text{SNR}_{\text{overlay}} = \frac{E_s\left(N - 2N_p\right)\Omega_c\sigma^2}{\left(N/\left(N - 2N_p\right)\right)N_0/2} \tag{63}$$

For fair comparison, the same SNR per bit is used for overlay and underlay methods in our evaluations. For the primary user the SNR is defined as $((E_p\Omega_c)/(N_0/2))$ in AWGN channel and the average SNR for fading channel is considered as $((E_p\Omega_c\sigma^2)/(N_0/2))$.

5.1 Performance analysis of primary users

In Fig. 5, we evaluated the performance of primary users in the presence of secondary users as interference. In overlay scheme using three level codes, there is no interference from secondary to primary users; thus, we investigate the system performance in underlay technique. We assumed that all secondary users have the same transmitted power as $((P_s)/(P_p)) = -16$ dB, where $P_p = ((E_p)/T)$ and $P_s = ((E_s)/T)$ are the transmitted powers of primary and secondary users, respectively. Fig. 5 shows that simulation results verify analytical evaluations. As expected, increasing the SNR

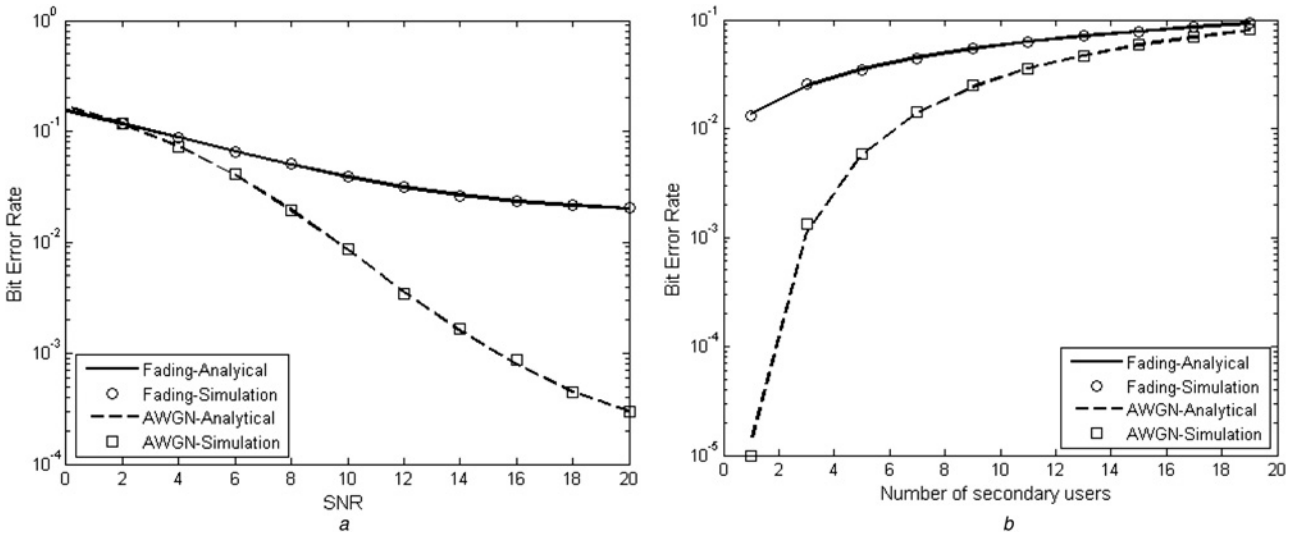

Fig. 5 *Performance of the desired primary user in underlay method for AWGN and fading channels, N = 128, ((Ps)/(Pp)) = −16 dB*
a $N_s = 3$
b SNR = 15 dB

$$\varphi_R(s) = \left[\left(\frac{1}{2}\right)^{N_s - 1}\sum_{k=0}^{N_s - 1}\binom{N_s - 1}{k}\frac{1}{2s\left(N_s - 2 - 2k\right)\sqrt{E_s}\Omega_c\sigma^2 - s^2\Omega_c\sigma^2\left((N_0)/2\right)\left(N/\left(N - 2N_p\right)\right) + 1}\right]^{\left(\left(N - 2N_p\right)/2\right)} \tag{61}$$

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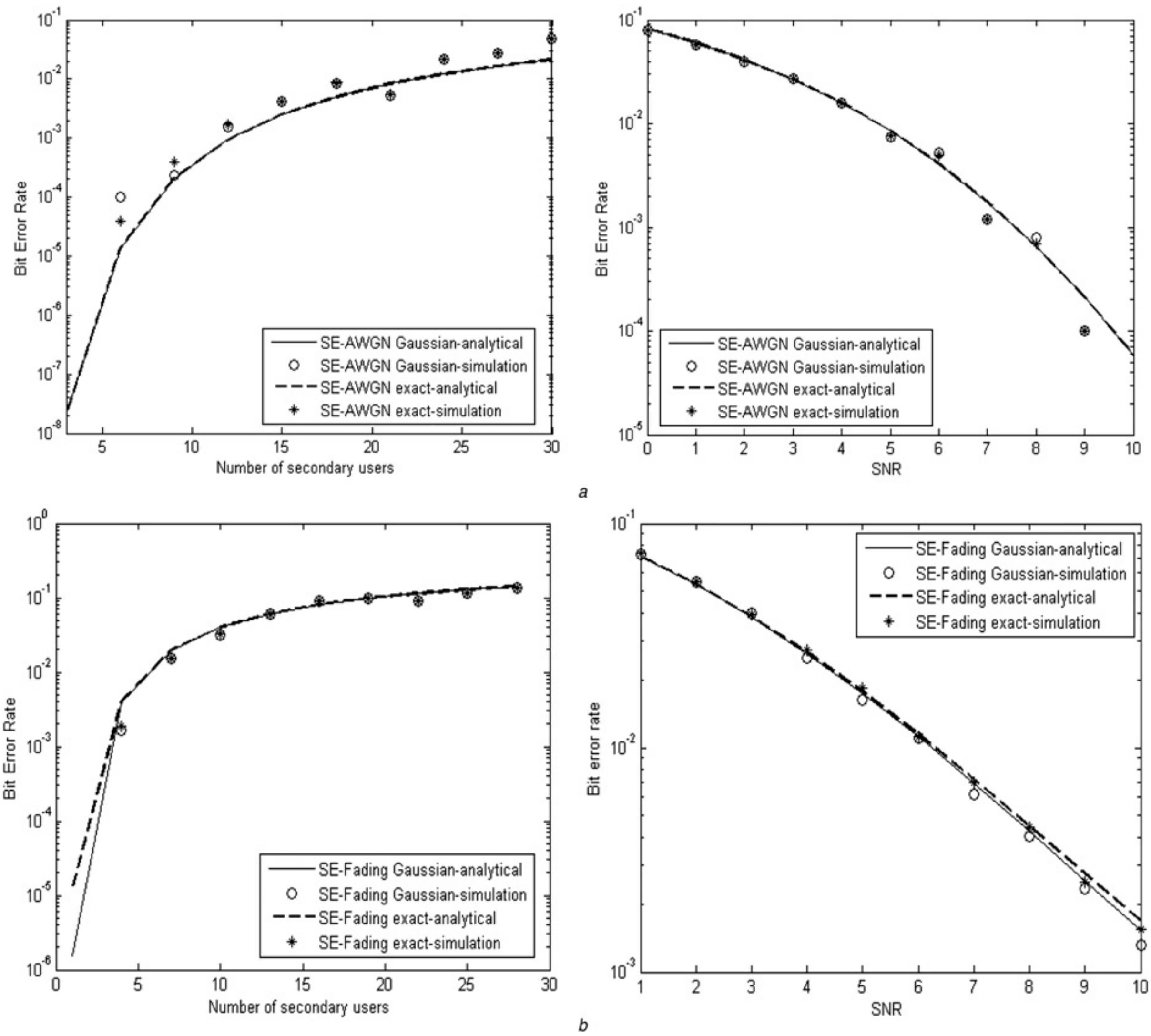

Fig. 6 *Performance of desired secondary user in AWGN and fading channels for both Gaussian assumption and exact analysis, $N_p = 3$, and $N = 128$*

a AWGN, underlay
b Fading, underlay

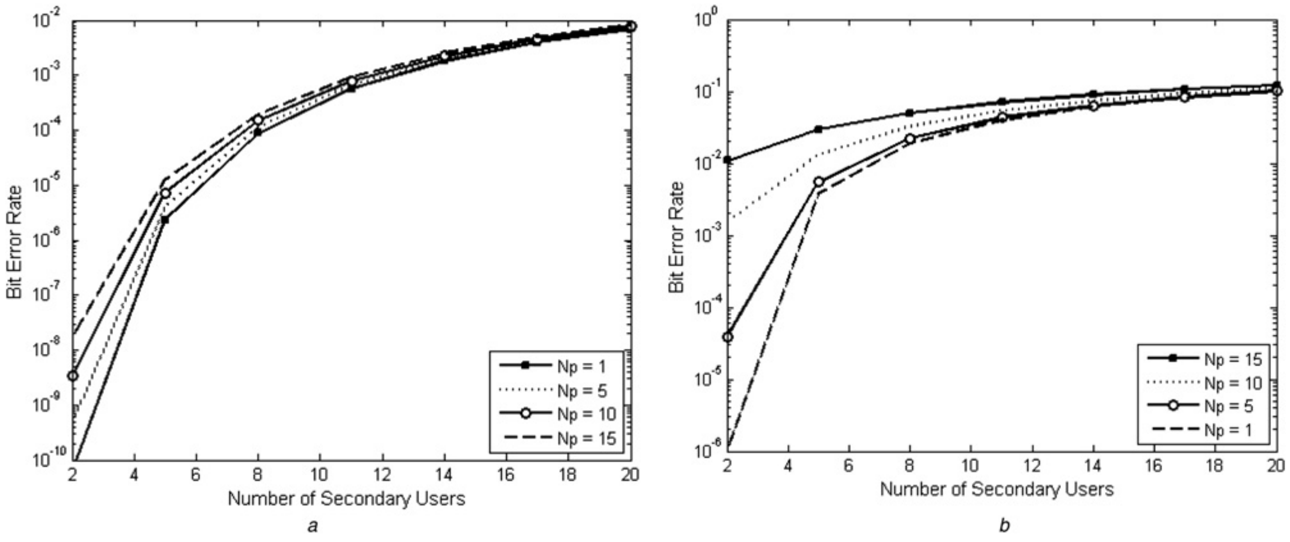

Fig. 7 *Performance of desired (first) secondary user in AWGN and fading channels for different number of primary users, $N = 128$, SNR = 15 dB and $((J_p)/(P_s)) = +10$ dB*

a AWGN, underlay
b Fading, underlay

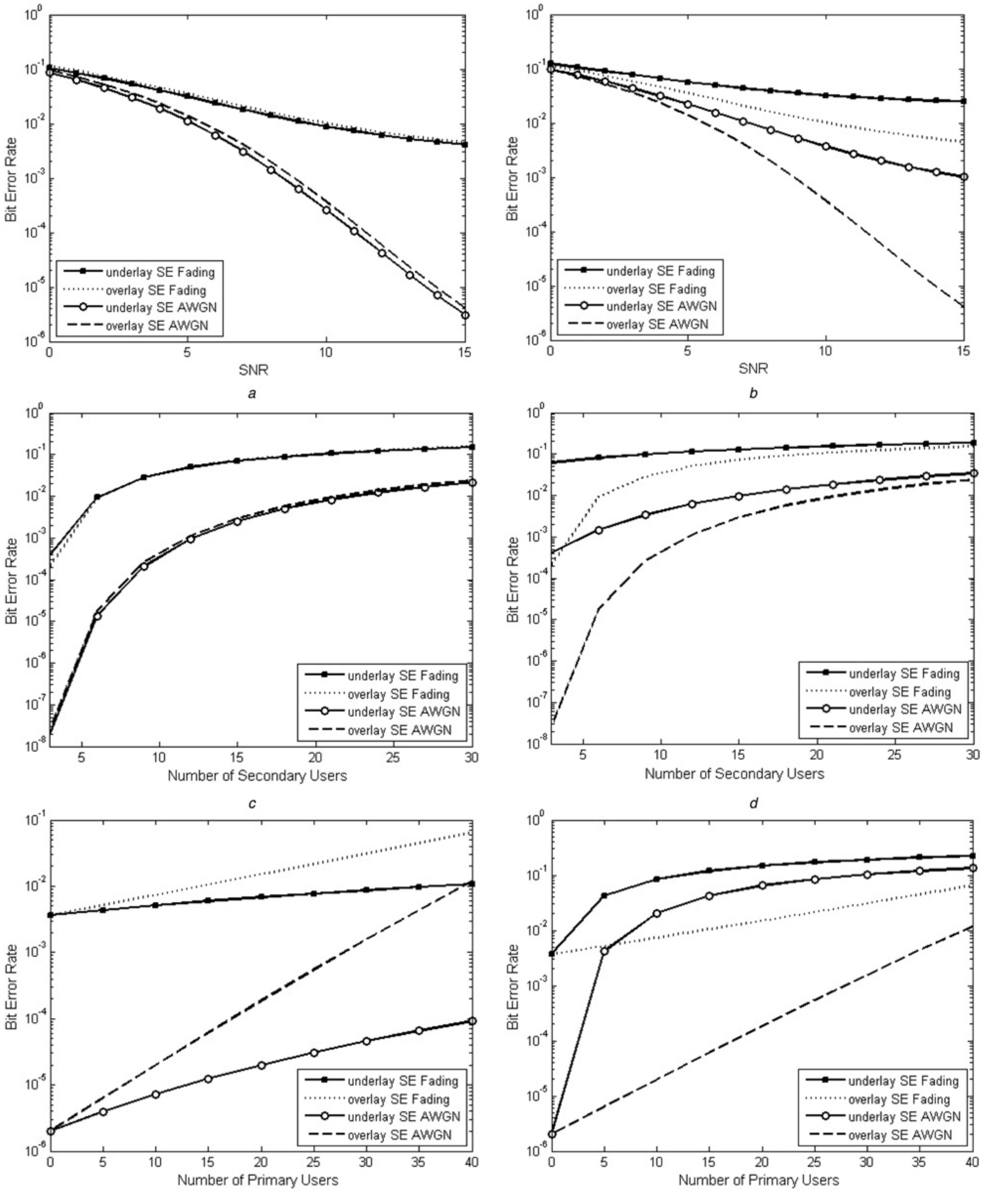

Fig. 8 *Performance comparison of underlay and overlay SE cognitive systems in AWGN and fading channels, N = 128*

a $((J_p)/(P_s)) = +10$ dB, $N_s = 5$, $N_p = 3$
b $((J_p)/(P_s)) = +25$ dB, $N_s = 5$, $N_p = 3$
c $((J_p)/(P_s)) = +10$ dB, SNR = 15 dB, $N_p = 3$
d $((J_p)/(P_s)) = +25$ dB, SNR = 15 dB, $N_p = 3$
e $((J_p)/(P_s)) = +10$ dB, SNR = 15 dB, $N_s = 5$
f $((J_p)/(P_s)) = +25$ dB, SNR = 15 dB, $N_s = 5$

reduces the probability of error. Additionally, it is observed from Fig. 5*b* that as the number of secondary users increases the system performance degrades. Since the interferences from the secondary users to the desired primary user increase and consequently this would raise the probability of error.

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5.2 Performance analysis of secondary users

Fig. 6 illustrates the performance of the desired (first) secondary user against the number of secondary users and SNR in AWGN and fading channels for both Gaussian assumption and exact analysis of NBI (due to the primary users) in the underlay approach. The interference-to-signal power ratios for both Gaussian assumption and exact analysis of NBI are set to 10 dB. We observe that when the number of secondary users increases, the probability of error increases in both AWGN and fading channels, because the interference from other secondary users increases. In addition, the increment of SNR will improve the system performance, as expected. It is observed that the simulation results verify analytical evaluations.

Further, as Fig. 6 shows in AWGN channel, the results for Gaussian assumption and exact analysis are the same. In fading channel, there is a little difference between the two curves in high SNRs or when the number of secondary users is less than five. As a result, considering the primary users NBIs as Gaussian processes is a reasonable assumption.

Fig. 7 shows the performance of desired (first) secondary user for different numbers of primary users in AWGN and fading channels. In overlay technique, the interference of primary users is omitted by assigning zero value codes in the frequency chips occupied by these users. As a result, we evaluate the effect of increase in the number of primary users for underlay scheme. All primary users have the same power spectrum. It is observed that as the number of primary users increases, the interference of primary users on the desired secondary user increases and the underlay performance degrades.

Now, we compare the underlay and overlay techniques for cognitive spectrally encoded system in different schemes. In Fig. 8, we have demonstrated that when the power spectrum of primary users is small, both underlay and overlay techniques have almost the same performance in terms of increasing SNR and number of secondary users (Figs. 8*a* and *c*), but when the power spectrum of narrowband primary users is large, overlay technique outperforms the underlay technique (Figs. 8*b* and *d*). The reason is that, when we assign zero code to the primary users' frequency bands in the overlay scheme, the interference from primary users to secondary users is cancelled. However, note that in overlay technique, we should increase the spectrum level in the chips that are not zero to compensate for the loss in signal power. In Figs. 8*e* and *f*, we compared both techniques in terms of increasing number of primary users. As shown, when the power spectrum of primary users is small, underlay technique has better performance for large number of primary users. When the number of primary users increases, the number of frequency chips that are assigned to zero in overlay scheme increases and this decreases the code length, available bandwidth and processing gain; consequently, the performance degrades. However, when the power spectrum of primary users is large, increasing the number of primary users degrades the performance of underlay technique more than overlay scheme.

The BER expression of desired secondary user in AWGN channel for SMSE framework is given in [3] by

$$P_b = Q\left(\sqrt{\text{SINR}}\right) = Q\left(\sqrt{\frac{2E_s}{2\sum_{l=1}^{N_p}\left(\left(M_l E_p^l\right)/\left(N_f\right)\right) + N_0}}\right) \tag{64}$$

where N_f is the number of frequency components for SMSE method. For fading channel, the BER is calculated in [4] as follows

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{E[\beta^2]E_s}{E[\beta^2]E_s + 2\sum_{l=1}^{N_p}\left(\left(M_l E_p^l\right)/\left(N_f\right)\right) + N_0}}\right) \tag{65}$$

The average SNR in [3, 4] is defined as

$$\text{SNR}_{\text{SMSE}} = \frac{E_s}{N_0/2} \tag{66}$$

For comparison, we assign $N_f = 64$ for SMSE technique as the

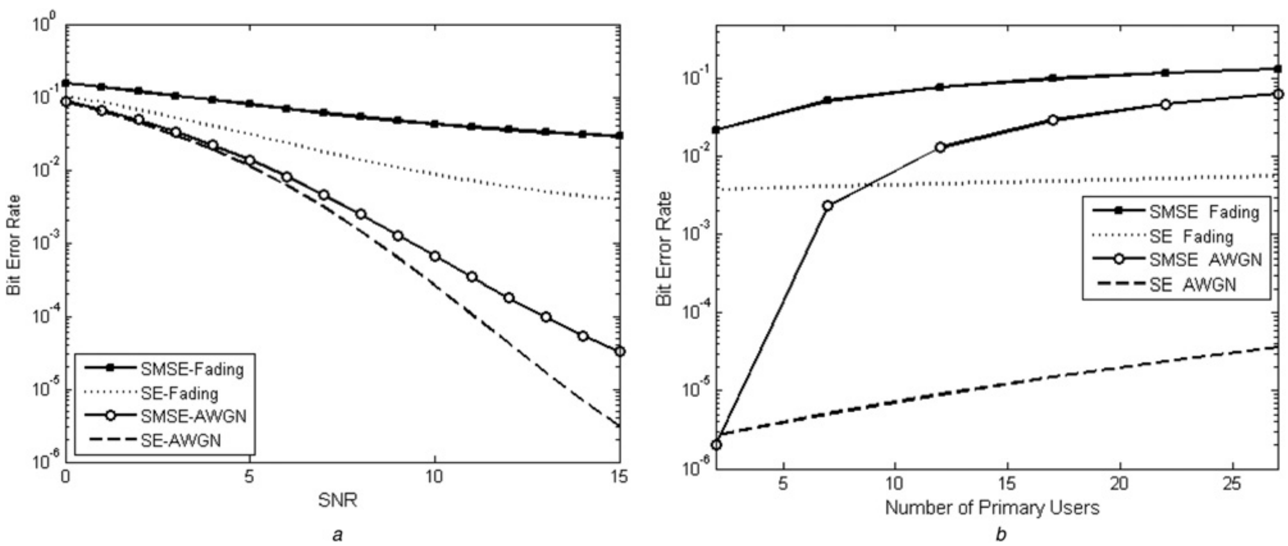

Fig. 9 *Performance comparison of cognitive underlay SE-CDMA and underlay SD-SMSE, $N_s = 5$, $N = 128$*

a $N_p = 3$, $((J_p)/(P_s)) = +10$ dB
b SNR = 15 dB, $((J_p)/(P_s)) = +10$ dB

number of frequency chips in SE method, $N/2 = 64$, and the number of subcarriers for OFDM is set $M_I = 1$ since we assumed that in our method each primary user occupies one frequency chip. The results illustrate that our proposed method significantly outperforms the SD-SMSE system (Fig. 9). This is because of flexibility of spectrally encoded CDMA in designing cognitive system parameters for overlay and underlay techniques.

6 Conclusions

In this paper, we studied the performance of multiple-access SE-CDMA in CR system with two approaches, overlay and underlay, in both AWGN and fading channels. Initially, we evaluated the performance of primary users with secondary users as interferences and then the performance of SE-CDMA secondary user in the presence of primary users as NBIs was obtained. The expressions for BER were calculated theoretically. It was shown that SE-CDMA has a great advantage in designing CR networks. SE-CDMA can utilise spectrum better than the other techniques by using codes in the frequency domain. In addition, it is flexible for designing the parameters of overlay and underlay methods. Hence, SE-CDMA is spectrally more efficient and it avoids the interference from primary users in the case of overlay approach. In the underlay approach, we considered the primary user effect as the NBI for secondary users and evaluated the performance of secondary user for both Gaussian assumption and exact analysis of primary users' interferences. The results demonstrated that the two cases had the same performance. For comparison, we also considered the results of SD-SMSE method proposed in [3, 4]. It was shown that our proposed method has better performance in comparison with SD-SMSE method. Future studies that suit real environment will consider the probability of false alarm and the probability of detection of the primary user presence.

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