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On the coding advantages of the quasi-orthogonal space–frequency block codes

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Abstract: In this study, the authors show that the coding advantage (CA) of quasi-orthogonal space–frequency block codes (QOSFBCs) could be decomposed into two parts, namely intrinsic CA (ICA) and extrinsic CA (ECA). Then, by using the ICA, the authors demonstrate that the optimum precoder of the QOSFBCs could be analytically derived without any requirement of the exhaustive search. Then, by improving the ECA, the authors enhance performances of the QOSFBCs efficiently – up to 4 dB. Finally, the authors show that a very fast optimisation process for the QOSFBCs is viable over any arbitrary frequency-selective channel.

1 Introduction

Space–time coding is one of the most advanced methods to combat the ruinous fading effect of the wireless channels for the narrowband wireless communication [1]. However, the intersymbol interference (ISI) effect of the wireless channels has emerged as an unwelcome challenge in the wideband wireless arena, which cannot be overcome by the existing space–time coding techniques alone. To tackle both the fading and the ISI effects, orthogonal frequency-division multiplexing (OFDM) and multiple-input multiple-output OFDM (MIMO-OFDM) have been developed.

Several space–frequency block codes (SFBC) have been proposed as an efficient method for implementing MIMO-OFDM systems, see [2–5] and references therein. Quasi-orthogonal SFBCs (QOSFBCs) are one of the most prosperous proposed SFBCs in the literature. The QOSFBCs are constructed based on the well-known quasi-orthogonal space–time block codes, and they benefit from a ‘simple decoding complexity’ in comparison with other existing SFBCs (see Section 6 of [3]).

In this paper, firstly, we show that the coding advantage (CA) of the QOSFBCs could be decomposed into two parts, namely intrinsic CA (ICA) and extrinsic CA (ECA). Secondly, by using the ICA, which depends on the constellation, precoder and the structure of the QOSFBCs, we demonstrate that the optimum precoder of the QOSFBCs could be analytically derived without any requirement of the exhaustive searches. Thirdly, by utilising the ECA, which depends only on the delay and power profiles (DPPs) of the channel and the applied permutation, we enhance performance of the QOSFBCs efficiently. More precisely, simulation results show the improvement in performance of the QOSFBCs up to 4 dB so that they outperform the recently proposed SFBCs. Finally, we discuss that a very fast optimisation process for the QOSFBCs is viable over any arbitrary frequency-selective channel.

Notations: We use boldface lowercase and boldface uppercase letters for vectors and matrices, respectively. Superscripts $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$ and $(\cdot)^{*}$ indicate transpose, Hermitian and complex conjugation, respectively. By $\circ$ and $\otimes$, we mean the Hadamard and the Tensor products, respectively. By $\lfloor\cdot\rfloor$, we mean the floor operation. $w_i \sim \mathcal{CN}(0, \sigma^2)$ denotes that w_i's $\forall i$ are independent identical distributed complex Gaussian random variable with zero-mean and variance σ^2. Notation $\mathbf{1}_a$ stands for an $a \times a$ matrix of ones, and $\boldsymbol{I}_a$ represents an identity matrix of size $a \times a$. Notations $\mathbb{N}$ and $\mathbb{C}$ stand for the natural numbers and the complex field, respectively. For matrix $\boldsymbol{A} \in \mathbb{C}^{m\times n}$, $\boldsymbol{A}(\alpha)$ stands for a submatrix of $\boldsymbol{A}$ which is obtained by sustaining rows of $\boldsymbol{A}$ indexed by the set α, and the principal submatrix of $\boldsymbol{A}$ is denoted by $\breve{\boldsymbol{A}}(\alpha)$ which lies in the rows and columns of $\boldsymbol{A}$ indexed by set α [6, p. 17]. Notation $\mathrm{diag}(a_1, a_2, \ldots, a_n)$ represents a diagonal $n \times n$ matrix whose diagonal entries are $a_1, a_2, \ldots, a_n$ and $V(\alpha_1, \alpha_2, \ldots, \alpha_K)$ denotes the $K \times K$ Vandermonde matrix.

2 System model

Let us define a codeword of the SFBC $\boldsymbol{C}$ as follows

$$C = \begin{bmatrix} c_1(0) & c_2(0) & \dots & c_{M_{\mathrm{T}}}(0) \\ c_1(1) & c_2(1) & \dots & c_{M_{\mathrm{T}}}(1) \\ \vdots & \vdots & \ddots & \vdots \\ c_1(N-1) & c_2(N-1) & \dots & c_{M_{\mathrm{T}}}(N-1) \end{bmatrix} \in \mathbb{C}^{N\times M_{\mathrm{T}}} \tag{1}$$

where M_T stands for the number of transmit antennas, N is the number of subcarriers per OFDM block and $c_i(n)$s are data transmitted by the ith transmit antenna at the nth subcarrier.

For a receiver with M_R antennas, in a frequency-selective channel with L independent delay paths between each pair of transmit and receive antennas, the channel impulse response from the transmit antenna i to the receive antenna j is given by

$$h_{(i,j)}(\zeta) = \sum_{(l=0)}^{(L-1)} \alpha_{i,j}(l)\delta\left(\zeta - \zeta_l\right) \quad (2)$$

where ζ_ls are delays and $\alpha_{i,j}(l) \sim \mathcal{CN}\left(0, \ \sigma_l^2\right)$ represents the complex amplitude corresponding to the lth path of the ith transmit and the jth receive antennas. Also we assume that there is no spatial fading correlation between antennas.

The received signal at the antenna j after matched filtering, removing the cyclic prefix and performing fast Fourier transform, at the nth frequency tone is given by

$$r_j(n) = \sum_{i=1}^{M_T} c_i(n)H_{i,j}(n) + \mathcal{N}_j(n), \quad n = 0, \ 1, \ \ldots, \ N-1 \quad (3)$$

In (3), $\mathcal{N}_j(n) \sim \mathcal{CN}\left(0, \ \sigma_n^2\right)$ stands for the additive white Gaussian noise, where σ_n^2 is the power of the noise and

$$H_{i,j}(n) = \sum_{l=0} \alpha_{i,j}(l)w^{n\zeta_l}, \quad n = 0, \ 1, \ \ldots, \ N-1 \quad (4)$$

is the channel frequency response at the nth frequency subcarrier between the transmit antenna i and the receive antenna j, and $w = e^{-j2\pi(BW/N)}$, where BW is the total bandwidth of the system.

3 Intrinsic and extrinsic CAs of the QOSFBCs

In this section, we show that the CA of the QOSFBCs could be decomposed into two independent parts, namely ICA and ECA. By using the ICA, which depends on the constellation, precoder and the structure of the QOSFBCs, we demonstrate that the optimum precoder of the QOSFBCs could be analytically derived without any requirement of the exhaustive searches explained in Section 3. Then, we show that both the ICA and the ECA are non-zero for any arbitrary DPPs indicating that the QOSFBCs are full-diversity for any arbitrary DPPs. It is worth mentioning that in [3], the authors have proven the full-diversity property of the QOSFBCs for the equal-power profile and integer delay taps.

As discussed in [2], if a SFBC is full-diversity, $\Delta^\circ \boldsymbol{R}_F$ is of rank LM_T, where $\Delta = \boldsymbol{V}\boldsymbol{V}^H$, $\boldsymbol{R}_F$ is the frequency correlation matrix and $\boldsymbol{V} = (\boldsymbol{C} - \tilde{\boldsymbol{C}})$ is the difference of two distinct codewords $\boldsymbol{C}$ and $\tilde{\boldsymbol{C}}$. The CA could then be defined as the product of all non-zero eigenvalues of $\Delta^\circ \boldsymbol{R}_F$. Now, we propose the following theorem:

Theorem: The CA of the QOSFBCs $\left(A_{QOSF}^C\right)$ can be written as the products of the ICA (ψ_{in}) and the ECA $\left(\psi_{ex}^{QOSF}\right)$, that is, $A_{QOSF}^C = \psi_{in}\psi_{ex}^{QOSF}$.

Proof: One can easily investigate that $\Delta^\circ \boldsymbol{R}_F$ associated with the QOSFBCs attains its minimum rank if blocks corresponding to two distinct codewords of $\boldsymbol{C}_{QOSF}$ differ only in one block, where $\boldsymbol{C}_{QOSF}$ a codeword of the QOSFBC with two transmit antennas. Without loss of generality, let this dissimilar block be $\boldsymbol{G}^1$. In this case, similar to what was argued in [3], it can be shown numerically that the minimum rank of the QOSFBCs is obtained when symbols $s_2^1, \ s_4^1, \ \ldots$ and s_{2L}^1 are the same in two distinct codewords. Thus, we have

$$A_{QOSF}^C = \det\left(\boldsymbol{V}_{non\text{-}zero}\boldsymbol{V}_{non\text{-}zero}^H \circ \boldsymbol{R}_F^{QOSF}\right) \quad (5)$$

In (5)

$$\boldsymbol{R}_F^{QOSF} = \boldsymbol{W} \ \mathrm{diag}\left(\sigma_0^2, \ \sigma_1^2, \ \ldots, \ \sigma_{(L-1)}^2\right)\boldsymbol{W}^H \quad (6)$$

where

$$\boldsymbol{W} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ w^{\zeta_0} & w^{\zeta_1} & \cdots & w^{\zeta_{L-1}} \\ \vdots & \vdots & \cdots & \vdots \\ w^{(2L-1)\zeta_0} & w^{(2L-1)\zeta_1} & \cdots & w^{(2L-1)\zeta_{L-1}} \end{bmatrix} \in \mathbb{C}^{2L\times L} \quad (7)$$

and $\boldsymbol{V}_{non\text{-}zero} \in \mathbb{C}^{2L\times 2}$ is defined below

$$\boldsymbol{V}_{non\text{-}zero} = \begin{bmatrix} \delta_1 & 0 & \delta_3 & \ldots & \delta_{2L-1} & 0 \\ 0 & \delta_1^* & 0 & \ldots & 0 & \delta_{2L-1}^* \end{bmatrix}^T \quad (8)$$

with

$$\left[\delta_1, \ \delta_3, \ \ldots, \ \delta_{2L-1}\right]^T = \boldsymbol{\Theta}\left[d_1, \ d_3, \ \ldots, \ d_{2L-1}\right]^T \quad (9)$$

and $d_i = s_i - u_i, \forall s_i, \ u_i \in \mathcal{A}$.

By defining $\boldsymbol{E} \in \mathbb{C}^{2L\times 2L}$ as

$$\boldsymbol{E} = \begin{bmatrix} \delta_1 & 0 & \delta_1 & \ldots & \delta_1 & 0 \\ 0 & \delta_1^* & 0 & \ldots & 0 & \delta_1^* \\ \delta_3 & 0 & \delta_3 & \ldots & \delta_3 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \delta_{2L-1} & 0 & \delta_{2L-1} & \ldots & \delta_{2L-1} & 0 \\ 0 & \delta_{2L-1}^* & 0 & \ldots & 0 & \delta_{2L-1}^* \end{bmatrix} \quad (10)$$

we can rewrite $\boldsymbol{V}_{non\text{-}zero}\boldsymbol{V}_{non\text{-}zero}^H$ as follows

$$\boldsymbol{V}_{non\text{-}zero}\boldsymbol{V}_{non\text{-}zero}^H = \boldsymbol{E} \circ \boldsymbol{E}^H \quad (11)$$

Therefore

$$A_{QOSF}^C = \det\left(\boldsymbol{V}_{non\text{-}zero}\boldsymbol{V}_{non\text{-}zero}^H \circ \boldsymbol{R}_F^{QOSF}\right) = \det\left(\boldsymbol{E} \circ \boldsymbol{E}^H \circ \boldsymbol{R}_F^{QOSF}\right) \quad (12)$$

Now, by using the properties of the Hadamard product and n-linearity of determinant function [6], we have

$$A_{QOSF}^C = \left(\prod\nolimits_{i=1,3,\ldots,2L-1} \left|\delta_i\right|^4\right) \det(\boldsymbol{\chi}) \quad (13)$$

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where $\chi = \left(\mathbf{1}_L \otimes \boldsymbol{I}_2\right) \circ \boldsymbol{R}_{\mathrm{F}}^{\mathrm{QOSF}}$. By defining

$$\psi_{\mathrm{in}} = \prod\nolimits_{i=1,\,3,\,\ldots,\,2L-1} |\delta_i|^4 \tag{14}$$

as the ICA and

$$\psi_{\mathrm{ex}}^{\mathrm{QOSF}} = \det(\boldsymbol{\chi}) \tag{15}$$

as the ECA, we can rewrite (13) as $A_{\mathrm{QOSF}}^{C} = \psi_{\mathrm{in}}\psi_{\mathrm{ex}}^{\mathrm{QOSF}}$. □

Now, let us discuss these two parts, that is, ψ_{in} and $\psi_{\mathrm{ex}}^{\mathrm{QOSF}}$, in more details

By expanding χ as follows

$$\boldsymbol{\chi} = (\mathbf{1}_L \otimes \boldsymbol{I}_2) \circ \left(\boldsymbol{W}\mathrm{diag}\left(\sigma_0^2,\ \sigma_1^2,\ \ldots,\ \sigma_{(L-1)}^2\right)\boldsymbol{W}^{\mathrm{H}}\right) \tag{16}$$

one could easily show that

$$\det(\boldsymbol{\chi}) = \det(\breve{\boldsymbol{\chi}}(\beta_o))\det(\breve{\boldsymbol{\chi}}(\beta_e)) \tag{17}$$

where $\beta_o = \{1, 3, \ldots, 2L-1\}$ and $\beta_e = \{2, 4, \ldots. 2L\}$. By defining $\boldsymbol{V}_o = \boldsymbol{W}(\beta_o)$, that is

$$\boldsymbol{V}_o = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ w^{2\zeta_0} & w^{2\zeta_1} & \cdots & w^{2\zeta_{L-1}} \\ \vdots & \vdots & \cdots & \vdots \\ w^{(L-1)2\zeta_0} & w^{(L-1)2\zeta_1} & \cdots & w^{(L-1)2\zeta_{L-1}} \end{bmatrix} \in \mathbb{C}^{L\times L} \tag{18}$$

we have

$$\breve{\boldsymbol{\chi}}(\beta_o) = \boldsymbol{V}_o \mathrm{diag}\left(\sigma_0^2,\ \sigma_1^2,\ \ldots,\ \sigma_{L-1}^2\right)\boldsymbol{V}_o^{\mathrm{H}} \tag{19}$$

and, therefore

$$\det(\breve{\boldsymbol{\chi}}(\beta_o)) = \prod_{(l=0)}^{(L-1)} \left(\sigma_l^2\right) \times |\det(\boldsymbol{V}_o)|^2 \tag{20}$$

It is obvious that $\boldsymbol{V}_o$ is a Vandermonde matrix and thus we have

$$\det(\breve{\boldsymbol{\chi}}(\beta_o)) = \prod_{l=0}^{L-1} (\sigma_l^2) \times \left|\prod\nolimits_{j<i} \left(w^{2\zeta_{i-1}} - w^{2\zeta_{j-1}}\right)\right|^2 \tag{21}$$

Similarly, we can show that

$$\det(\breve{\boldsymbol{\chi}}(\beta_e)) = \prod_{(l=0)}^{(L-1)} (\sigma_l^2) \times \left|\prod\nolimits_{(j<i)} \left(w^{\zeta_{i-1}} - w^{\zeta_{j-1}}\right)\right|^2 \tag{22}$$

Since $\zeta_0 < \cdots < \zeta_{L-2} < \zeta_{L-1}$, $\prod_{j<i}\left(w^{2\zeta_{i-1}} - w^{2\zeta_{j-1}}\right)$ and $\prod_{j<i}\left(w^{\zeta_{i-1}} - w^{\zeta_{j-1}}\right)$ are non-zero [2].

Remark 1: The ECA, as (9) and (13) indicate, depends only on the precoder matrix $\boldsymbol{\Theta}$. As discussed in Section 3, the precoder proposed in [3] is a Hadamard matrix multiplied by a parametric diagonal matrix for rotating the constellation. The problem of this precoder is that it needs an exhaustive search for each constellation. Now, using the ICA, we could formulate an optimum precoder for the QOSFBCs. Fortunately, (14) has a familiar form known as the minimum product distance in the literature. So far many linear constellation precoders have been proposed for this problem, the Vandermonde precoder is known as one of the best linear constellation precoders for lattice constellation points [7]: $\boldsymbol{\Theta} = (1/\lambda)V(\alpha_1, \alpha_2, \ldots, \alpha_K)$ with $\{\alpha_m\}_{m=1}^{K}$ are the roots of $m_{\alpha,\,\mathbb{Q}(j)}(x)$ and $1/\lambda$ is for normalisation, where $\mathbb{Q}(j)$ denotes the smallest subfield of $\mathbb{C}$ including both $\mathbb{Q}$ and j. Also $m_{\alpha,\,\mathbb{F}}(x)$ denotes the minimal polynomial of α over a field $\mathbb{F}$ and $\deg\left(m_{\alpha,\,\mathbb{Q}(j)}(x)\right) = K$.

Remark 2: Since the ICA and the ECA of the QOSFBCs are non-zero, $\boldsymbol{A}_{\mathrm{QOSF}}^{C}$ is non-zero and the QOSFBCs are full-diversity for any arbitrary DPPs.

4 Proposed modified QOSFBCs

It is well investigated that the permutation is very effective in enhancing the performance of SFBCs [2, 4]. In this section, regarding the CA decomposition discussed in the previous section, we introduce a permutation parameter to the structure of the QOSFBCs. Note that adding the permutation can be performed unrelatedly to the CA decomposition. At the end of this section we show that how decomposing the CA could lead to a very low complexity computation of permutation parameter.

The QOSFBCs proposed in [1, 3] do not have the freedom of choosing the diversity and consequently receiver complexity. Here, we represent the modified versions of QOSFBCs with the diversity order of 2Γ $(1 \le \Gamma \le L)$ as

$$\boldsymbol{C}_{\text{P-QOSF}} = \boldsymbol{P}\boldsymbol{C}_{\mathrm{QOSF}} \tag{23}$$

where $= \mathrm{diag}\left(\boldsymbol{P}_t,\ \boldsymbol{I}_b\right) \in \mathbb{N}^{N\times N}$, and

$$\boldsymbol{P}_t = \mathrm{diag}\left(\overbrace{\boldsymbol{P}_b,\ \boldsymbol{P}_b,\ \ldots,\ \boldsymbol{P}_b,}^{\text{Number of } \boldsymbol{P}_b\text{s} = \left|\frac{N}{\Gamma\gamma_{\mathrm{SD}}}\right|}\quad \boldsymbol{P}_b'\right) \otimes \boldsymbol{I}_{M_t} \tag{24}$$

In (24), γ_{SD} is the permutation parameter that will be indicated later, and

$$\boldsymbol{P}_b = \left[\boldsymbol{P}_1^{\mathrm{T}}\quad \boldsymbol{P}_2^{\mathrm{T}}\quad \cdots\quad \boldsymbol{P}_\Gamma^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{N}^{\Gamma((\gamma_{\mathrm{SD}})/(M_t))\times\Gamma((\gamma_{\mathrm{SD}})/(M_t))}$$

where

$$\boldsymbol{P}_i = \left[\boldsymbol{e}_i^{\mathrm{T}}\quad \boldsymbol{e}_{\Gamma+i}^{\mathrm{T}}\quad \cdots\quad \boldsymbol{e}_{(((\gamma_{\mathrm{SD}})/(M_t))-1)\Gamma+i}^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{N}^{((\gamma_{\mathrm{SD}})/(M_t))\times\Gamma((\gamma_{\mathrm{SD}})/(M_t))} \tag{25}$$

For $i = 1, 2, \ldots, \Gamma$, with $\boldsymbol{e}_j \in \mathbb{C}^{1\times\Gamma((\gamma_{\mathrm{SD}})/(M_t))}$ and $\boldsymbol{e}_j' \in \mathbb{C}^{1\times\Gamma\lfloor((\gamma_r)/(\Gamma M_t))\rfloor}$ are vectors whose components are all zeros except for the jth element that is one, and

$$\boldsymbol{P}_b' = \left[\boldsymbol{P}_1'^{\mathrm{T}}\quad \boldsymbol{P}_2'^{\mathrm{T}}\quad \cdots\quad \boldsymbol{P}_{\lfloor((\gamma_r)/(\Gamma M_t))\rfloor}'^{\mathrm{T}}\right]^{\mathrm{T}} \in C^{\Gamma\lfloor((\gamma_r)/(\Gamma M_t))\rfloor\times\Gamma\lfloor((\gamma_r)/(\Gamma M_t))\rfloor} \tag{26}$$

where $\gamma_r = N - \Gamma\gamma_{SD}\lfloor (N/(\Gamma\gamma_{SD}))\rfloor$ and for $i = 1, 2, \ldots, \lfloor ((\gamma_r)/(\Gamma M_t))\rfloor$

$$\boldsymbol{P}'_i = \left(\boldsymbol{e}_i'^{\mathrm{T}} \quad \boldsymbol{e}'^{\mathrm{T}}_{\lfloor ((\gamma_r)/(\Gamma M_t))\rfloor + i} \quad \cdots \quad \boldsymbol{e}'^{\mathrm{T}}_{(\Gamma-1)\lfloor ((\gamma_r)/(\Gamma M_t))\rfloor + i} \right)^{\mathrm{T}} \in C^{\Gamma \times \Gamma \lfloor ((\gamma_r)/(\Gamma M_t))\rfloor} \tag{27}$$

also $\boldsymbol{I}_b$ is an identity matrix of size $(\gamma_r - \Gamma M_t \lfloor ((\gamma_r)/(\Gamma M_t))\rfloor) \times (\gamma_r \Gamma M_t \lfloor ((\gamma_r)/(\Gamma M_t))\rfloor)$.

In what follows, we discuss how the proposed permuted QOSFBCs (P-QOSFBCs) could enhance the performance of the QOSFBCs. By doing operations similar to (5)–(14), one can readily obtain the CA of the P-QOSFBCs as follows

$$A^{C}_{\text{P-QOSF}} = \psi_{\text{in}} \psi^{\text{P-QOSF}}_{\text{ex}} \tag{28}$$

where

$$\psi^{\text{P-QOSF}}_{\text{ex}} = \det\left(\left(\boldsymbol{1}_L \otimes \boldsymbol{I}_2\right) \circ \boldsymbol{R}^{\text{P-QOSF}}_{\text{F}} \right)$$

and $\boldsymbol{R}^{\text{P-QOSF}}_{\text{F}} \in \mathbb{C}^{2L \times 2L}$ is shown below (see (29))

Equation (28) shows that the CA of the P-QOSFBCs depends on both the ICA and ECA. From (29), it can be understood that the ECA part $\left(\psi^{\text{P-QOSF}}_{\text{ex}}\right)$ depends on γ_{SD} as well as on the system bandwidth (BW), DPPs and the number of subcarriers. Therefore, if DPPs are known to the transmitter, we can find γ_{SD} so as to maximise the CA of the P-QOSFBCs. On the other hand, if DPPs are unknown to the transmitter, we optimise the CA of the P-QOSFBC based on the artificial DPPs (ADPPs) that we have introduced in [5]. In short, we could formulate the problem of finding the optimum positive integer $\gamma^{\text{OP}}_{\text{SD}}$ to maximise the CA of the P-QOSFBCs as

$$\gamma^{\text{OP}}_{\text{SD}} = \arg \max_{1 < \gamma_{\text{SD}} < \left\lfloor \frac{N}{\Gamma} \right\rfloor} \det\left(\left(\boldsymbol{1}_L \otimes \boldsymbol{I}_2\right) \circ \boldsymbol{R}^{\text{P-QOSF}}_{\text{F}} \right) \tag{30}$$

It is worthwhile to mention that clearly the optimisation process is the calculation of $(((\lfloor N/\Gamma \rfloor)/2) - 2)$ determinants of matrices of size $\Gamma M_t \times \Gamma M_t$, which imposes a very low computational complexity on the transmitter compared with the typical optimisation processes. It is also obvious that the complexity of the maximum-likelihood decoder of the P-QOSFBCs is the same as those of the QOSFBCs and the block circular delay diversity (BCDD) codes [4].

Remark 3: Now, let us discuss why derivation of (30) is significantly useful. Obtaining optimum permutation for QOSFBCs among N_γ possible permutations, using the $A^{C}_{\text{P-QOSF}}$ as the objective function, means that $N_\gamma M^{N_s}$ determinants of $\Gamma M_t \times \Gamma M_t$ matrices must be calculated, where $N_s = \Gamma M_t$ and M is the size of used constellation. Whereas with the $\psi^{\text{P-QOSF}}_{\text{ex}}$, it only requires calculating N_γ determinants. In other words, the computation complexity of order $\mathcal{O}\left(T \times M^{N_s}\right)$ decreases as $\mathcal{O}(T)$, where $T = N_\gamma (\Gamma M_t)^3$. To illustrate this point, we consider an example of a practical MIMO-OFDM system. For a MIMO-OFDM system with two transmit antennas and $N_c = 128$ subcarriers in the six-ray typical urban channel model [8], optimising parameter γ_{SD} of the P-QOSFBCs using the $A^{C}_{\text{P-QOSF}}$ for 16-QAM constellation calls for the lengthy calculation of $N_\gamma M^{N_s} = 10{*}16^{2*6} \simeq 10^{15}$ determinants of 12×12 matrices, that is, complexity order of $\mathcal{O}\left(10^{18}\right)$. In contrast, if the (30) is used for maximising $\psi^{\text{P-QOSF}}_{\text{ex}}$ of P-QOSFBC, only $N_\gamma = 10$ determinants of 12×12 matrices needs to be computed, that is, the complexity order is $\mathcal{O}\left(10^4\right)$.

Remark 4: For the P-QOSFBCs, we made a trade-off between the diversity advantage (which is equal to $\Gamma M_t M_r$) and the receiver complexity of the ML decoder (which is in the order of $\mathcal{O}\left(M^{M_t \Gamma}\right)$). This trade-off is controlled by selecting the value of Γ. More precisely, if $\Gamma = L$ (resp. $\Gamma = 1$), the P-QOSFBCs achieve the maximum (resp. minimum) attainable diversity advantage equal to $L M_t M_r$ (resp. $M_t M_r$) and the most (resp. least) receiver complexity in the order of $\mathcal{O}\left(M^{(M_t L)}\right)$ (resp. $\mathcal{O}\left(M^{M_t}\right)$).

5 Simulation results

In our simulations, we consider a MIMO-OFDM space–frequency coded system with the following characteristics and parameters: one receive and two transmit antennas, total BW of 1 MHz, cyclic prefix of length 20 µs and 2-ray equal-power Rayleigh channels between each pair of transmit and receive antennas.

In Figs. 1 and 2, it is assumed that DPPs are unknown to the transmitter. In this case, ADPPs result in $\gamma^{\text{OP}}_{\text{SD}}$ equal to 10 and 78 for the number of subcarriers equal to 128 and

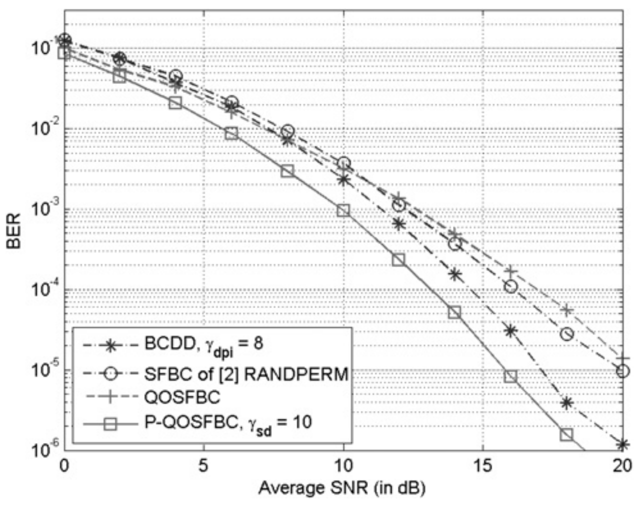

Fig. 1 *BER performance, N = 128, delay spread 5 µs, 1 bit/s/Hz*

$$\begin{bmatrix} 1 & \sum_{l=0}^{L-1} \sigma_l^2 w^{-\gamma_{\text{SD}}\zeta_l} & \cdots & \sum_{l=0}^{L-1} \sigma_l^2 w^{-(2L-1)\gamma_{\text{SD}}\zeta_l} \\ \sum_{l=0}^{L-1} \sigma_l^2 w^{\gamma_{\text{SD}}\zeta_l} & 1 & \cdots & \sum_{l=0}^{L-1} \sigma_l^2 w^{-(2L-2)\gamma_{\text{SD}}\zeta_l} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{l=0}^{L-1} \sigma_l^2 w^{(2L-1)\gamma_{\text{SD}}\zeta_l} & \sum_{l=0}^{L-1} \sigma_l^2 w^{(2L-2)\gamma_{\text{SD}}\zeta_l} & \cdots & 1 \end{bmatrix} \tag{29}$$

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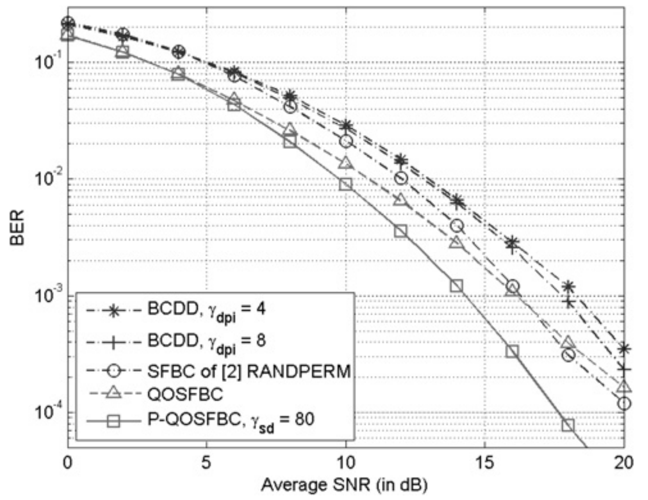

Fig. 2 *BER performance, N = 1024, delay spread 10 µs, 2 bits/s/Hz*

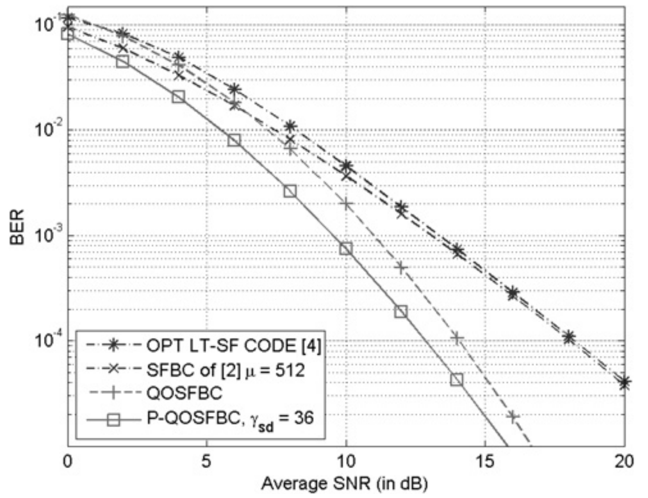

Fig. 3 *BER performance, N = 1024, delay spread 15 µs, 1 bit/s/Hz*

1024, respectively. The values of γ_{SD}^{OP} are numerically calculated using (30). On the other hand, Fig. 3 shows the simulation results for the known DPPs. In this case, γ_{SD}^{OP} is obtained as 36 for the P-QOSFBCs. Also for the optimum SFBC proposed in [4], θ and γ_{dpi} are optimised as 2 and 256, respectively, for $N = 1024$ and the 2-ray equal-power channel with delay spread 15 µs.

For the SFBCs proposed in [2], the random permutation is used in the simulations of Figs. 1 and 2 [2]. For the known DPPs, the parameter μ associated with the proposed code in [2] is 256.

As Figs. 1–3 show, in all cases, the proposed P-QOSFBCs outperform the QOSFBCs effectively and exhibit superior performances compared with the SFBCs proposed in [2], the BCDD codes, and also the optimum SFBC proposed in [4], which are the best existing SFBCs in the literature to the best of author's knowledge.

For example, from Fig. 1, at a bit error rate (BER) = 10^{-5} and for the channel with delay spread of 5 µs, the P-QOSFBCs achieve about 4 and 1 dB gains over the QOSFBCs and the BCDD codes, respectively. The obtained CAs displayed in Table 1 also are in accordance with the simulation results of Figs. 1–3.

Table 1 CAs of different SFBCs

	[5]	QOSFBC	P-QOSFBC
$N = 128$ (BPSK)	7.647	0.088	19.647
$N = 1024$ (QPSK)	2.24×10^{-4}	6.5×10^{-7}	0.007
$N = 1024$ (BPSK)	0.0074	0.0018	24.633

BPSK, binary phase-shift keying; QPSK, quadrature phase shift keying

6 Conclusion

In this paper, we showed that the CA of the QOSFBCs could be decomposed into two parts, called intrinsic and extrinsic CAs. Using this decomposition, we could analytically formulate the precoders of the QOSFBCs. We also proposed a modified version of the QOSFBCs, and demonstrated how the optimisation process could be simply and swiftly done for the QOSFBCs by using the aforementioned parts of the CA. Simulation results illustrate a superior performance of our proposed modified versions of the QOSFBCs in comparison with the recently proposed SFBCs.

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