Published in IET Communications
Received on 10th June 2008
Revised on 14th December 2008
doi: 10.1049/iet-com.2008.0368

ISSN 1751-8628

Unified approach to the capacity evaluation of the relay channel

G.A. Hodtani M.R. Aref
Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran
E-mail: hodtani@yahoo.com

Abstract: The authors indicate the dependence between the inputs of the relay channel with one auxiliary random variable as Cover, El-Gamal and Salehi have done for the multiple access channel with arbitrarily correlated sources. Then, by considering broadcast and multiple access sub-channels in the relay channel, the authors describe the essential role of the relay with special Markovity conditions on the auxiliary random variable and channel input–outputs, and unify most of known capacity theorems into one capacity theorem. The capacity theorem potentially may be applicable to a more general class of relay channels including at least the relay channels with known capacity.

1 Introduction

1.1 The relay channel

The discrete and memoryless relay channel (Fig. 1) consists of four finite sets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}, \mathcal{Y}_1$ and a collection of probability distributions $p(\cdot, \cdot|x_1x_2)$ on $\mathcal{Y} \times \mathcal{Y}_1$, one for every $(x_1, x_2) \in \mathcal{X}_1 \times \mathcal{X}_2$; y and y_1 are the channel outputs and are received by the receiver and the relay, respectively; x_1 and x_2 are the channel inputs and are sent by the transmitter and the relay.

An $(2^{nR}, n)$ code for the relay channel consists of a set of integers $\mathcal{M} = \{1, 2, \ldots, 2^{nR}\}$, an encoding function that maps each message $w \in \mathcal{M}$ into a code word x_1, $x_1{:}\mathcal{M} \rightarrow x_1^n$, a set of relay functions $\{f_i\}_{i=1}^n$ such that $x_{2i} = f_i\{y_{11}, y_{12}, \ldots, y_{1i-1}\}$, $1 \le i \le n$ and a decoding function $g: y^n \rightarrow \mathcal{M}$. A rate R is achievable if there exists a sequence of $(2^{nR}, n)$ codes with $P_e^{(n)} = P\{\hat{w} \neq w\} \rightarrow 0$ as $n \rightarrow \infty$. Channel capacity C is defined as the supremum over the set of achievable rates.

1.2 Background

The relay channel was first introduced by Van der Meulen [1]. In [2] the capacity of degraded and reversely degraded relay channels and the capacity of the relay channel with feedback as well as upper and lower bounds on the capacity of the general relay channel were established. In [3] the capacity of semi-deterministic relay channel, in [4, 5] the capacity of relay channel with orthogonal components, in [6] the capacity of modulo-sum relay channel (MSRC) and in [7] the capacity of a class of deterministic relay channels have been determined. The capacity of the general relay channel is still unknown; therefore the challenge on the relay channel is to work about the problem of the capacity.

In most known capacity theorems ([2, degraded and full feedback relay channels], [3, semi-deterministic relay channel], [4, orthogonal relay channel]) the rate is achieved using decode-and-forward (DAF) strategy [2, 8] and the cut-set bound achieving converse is proved under the restrictions imposed to the definition of the special channel. All of these capacities cannot be achieved via estimate-and-forward (EAF) strategy [2, 8] and among those, we have only the reversely degraded relay channel, the capacity of which can be achieved by both DAF and EAF strategies (Section 6).

1.3 The relay channel and the multiple access channel

In multiple access channel with correlated sources, the channel inputs are dependent and in [9] this dependence has been indicated by three auxiliary random variables and to allow partial co-operation between the transmitters, the codes are allowed to depend statistically on the source outputs. In the relay channel, there is only one source and the channel inputs are dependent; we can indicate this dependence with one auxiliary random variable and allow the codes to depend on the source outputs as in [9].

1.4 Our work

Here, we define almost general relay channel with decode-and-forward (AGRCDAF) strategy and determine its capacity. Then we show that all of the relay channels with known capacity are special cases of it. We have unified the previous capacity theorems into one theorem towards clarifying, helping and co-operating role of the relay in the relay channel and in all of these theorems. Our theorem is applicable at least to those relay channels that satisfy the constraints of the theorem. DAF strategy is one of many possible coding schemes and the best scheme till date. Of course, we do not claim that it is optimal for any relay channel.

Also, we show that our capacity theorem is consistent with the capacity regions for broadcast channel with degraded message sets and the multiple access channel with partially co-operating encoders (in a special case).

1.5 Paper organisation

The paper is organised as follows. In Section 2, we categorise the relay channels into two groups. In Section 3, the role of the relay is described mathematically and in Section 4, we define AGRCDAF, that includes interpretation, the definition and explanation of the properties of AGRCDAF. In Section 5, the capacity of the AGRCDAF is determined and proved. In Section 6, we show that the known capacity theorems are special cases of our capacity theorem and also validate our capacity theorem by its consistency with previous results regarding special broadcast channel and multiple channel with co-operating encoders (in a special case). Finally, the conclusion in Section 7.

2 Two important groups of the relay channels

The inputs X_1X_2 in Fig. 1 can be dependent or independent. One auxiliary random variable U, which is related to X_1X_2 by a distribution function $p(x_1x_2u)$, can indicate the dependence of the inputs. In this case the relay can decode U of X_1 through X_2Y_1 ($U \neq \emptyset$). When the inputs are independent, the relay estimates $\hat{Y}_1$ of $Y_1(U = \emptyset)$ [2].

To date, two important coding strategies or the combination of them have been applied to the relay channels to obtain the best rates in different situations [2, 8]. Therefore we can categorise the relay channels into two groups: the relay channels with DAF strategy ($U \neq \emptyset$); and the relay channels with EAF strategy ($U = \emptyset$).

2.1 The relay channels with DAF strategy

DAF strategy is one of many possible coding schemes (although the best scheme up to now) that gives the best rate when the channel between the sender and the relay is a good one. To date the best rate of DAF strategy is the rate in [2, theorem 7, $\hat{Y}_1 = \emptyset$, $V = X_2$, $U = (X_2, U)$] for the relay channel in Fig. 1

$$R = \sup_{p(x_2x_2u)}\min\{I(X_1X_2; Y), \\ I(U; Y_1|X_2) + I(X_1; Y|X_2U)\} \quad (a_1)$$

where supremum is taken over all by $p(x_1x_2u)$ and U is an auxiliary random variable.

2.2 The relay channels with EAF strategy

EAF strategy gives the best rate when the channel between the relay and the receiver has a better quality. To date the best rate obtained by this strategy is the rate R_1^* in [2, theorem 6]

$$R_1^* = \sup I(X_1; Y\hat{Y}_1|X_2)$$

subject to the constraint

$$I(X_2; Y) \geq I(Y_1; \hat{Y}_1|X_2Y)$$

where the supremum is taken over all joint distribution on $\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{Y} \times \mathcal{Y}_1 \times \hat{\mathcal{Y}}_1$ of the form

$$p(x_1x_2\,y\,y_1\,\hat{y}_1) = p(x_1)p(x_2)p(yy_1|x_1x_2)p(\hat{y}_1|y_1x_2) \quad (a_2)$$

and $\|\hat{Y}_1\| < \infty$, where $\|\cdot\|$ indicates the cardinality.

In [10] for the noisier relay-receiver channel, that is

$$I(\hat{Y}_1; Y_1|X_2Y) - I(\hat{Y}_1; X_1|X_2Y) \leq I(X_2; Y) \leq I(\hat{Y}_1; Y_1|X_2Y)$$

the following rate R_2^* (the rate less then R_1^*) is achieved

$$R_2^* = \sup\{I(X_1; Y|X_2) - H(\hat{Y}_1|X_2YX_1) + I(X_2; Y) \\ + H(\hat{Y}_1|X_2Y_1)\}$$

where supremum is taken over by (a_2).

In [11] with EAF strategy, a rate has been obtained when the channel inputs are block internally dependent instead of the independence of the inputs in (a_2).

3 Mathematical description of the relay

3.1 Mathematical description of the relay's role

The main idea of Van der Meulen [1] in introducing the relay channel and the main explanation in [2] about the relay channel are as follows.

When Y (Fig. 1) cannot communicate with X_1, the relay Y_1 can co-operate with Y in making the communication possible and or increasing the rate of data transmission, namely, Y_1 along with X_2, can decode some information from the message sent to Y by X_1, and then transmit it to Y by X_2. The receiver, upon receiving Y, can find X_2 and then decode the information understood and transmitted by the relay, thereby understanding X_1.

Since [1, 2], thus far, the relay channel has been described statistically by probability distributions $p(yy_1|x_1x_2)$. This description may be any observation of statistical behaviour of the channel and can also show other channels, such as two way and interference channels. In all of these descriptions, the helping role of the relay and the co-operation between the sender and the relay, in other words, the logical foundations of the relay's definition are not seen clearly.

In [9] the dependence between the inputs of the multiple access channel with correlated sources has been shown using auxiliary random variables and here, similarly, we can reveal the relay's role and describe the correlation between channel inputs by one auxiliary random variable U, the mathematical relationship of which to $(X_1X_2YY_1)$, for any relay channel, can be stated as

$$p(x_1x_2uyy_1) = p(x_1x_2u)p(yy_1|x_1x_2) \qquad \text{(i)}$$

where, U is related to YY_1 through the co-operation between X_1X_2 and X_1X_2 are related to U with $p(x_1x_2u)$. Then, we can axiomise the essential property of the relay and explain mutual relations of $(X_1X_2YY_1U)$, as follows.

3.2 Mathematical description of the relay's co-operation

For the relay we have two cases:

Case 1: The relay co-operates with the sender and is not useless. In this case:

First, the relay (X_2Y_1) understands U of X_1. Mathematically $X_1 \to U \to X_2Y_1$ or necessarily

$$X_1 \to X_2U \to Y_1 \qquad \text{(ii)}$$

Second, the relay must be better than the receiver in finding U. Mathematically

$$U \to X_2Y_1 \to Y \qquad \text{(iii)}$$

Case 2: The relay is useless, that is, the receiver is itself better than the relay in finding U and X_1 (sufficiently $UX_1 \to Y \to Y_1X_2$ or necessarily from (6) below $X_1 \to X_2UY \to Y_1$, or sufficiently the condition (ii) and the accompanying condition

$$Y_1 \to X_1X_2U \to Y \qquad \text{(iv)}$$

So, we can define a more general class of relay channels, considering (ii), (iii) and (iv), as follows in Section 4.

4 Almost general relay channel with DAF strategy

Auxiliary random variables have been used in achievability results (interference channels [12], Z-channels [13], cognitive radio channels [14], broadcast channels [15 and references therein], multiple access channels [9, 16], relay channels [2]), in capacity results (broadcast channels [15 and references therein], relay channels [7], side information channels [17], multiple access channels [9, 18]) and in defining channels (less noisy broadcast channels [19], modified interference channels [12], modified cognitive radio channels [14]). Here we use one auxiliary random variable in the definition and the capacity of a class of relay channels.

As mentioned in the introduction, most of known capacity theorems ([2, degraded and full feedback relay channels], [3, semi-deterministic relay channel], [4, orthogonal relay channel]) have their achievability parts by DAF strategy and have their cut-set bound achieving converses under the restrictions imposed to the definition of the special channel. Now, having considered this common property for known capacity theorems and also the explanations in Section 3, we can gather all these as Markovity conditions consistent with logical foundations of the definition of the relay channel.

Definition: A discrete and memoryless relay channel $p(yy_1|x_1x_2)$ (Fig. 1) is said to be AGRCDAF strategy if there exists joint distribution $p(x_1x_2u)$ such that

$$X_1 \to X_2U \to Y_1 \qquad (1)$$

$$U \to X_2Y_1 \to Y \qquad (2)$$

$$Y_1 \to X_1X_2U \to Y, \qquad U \neq X_1 \qquad (3)$$

where

$$p(x_1x_2uyy_1) = p(x_1x_2u)p(yy_1|x_1x_2) \qquad (4)$$

and U is one auxiliary random variable in all (1)–(3) and represents the information decodable by the relay, of X_1 through Y_1 and X_2.

Remark: In this definition, $U =$ constant does not represent the decodable information and therefore it is not and need

doi: 10.1049/iet-com.2008.0368

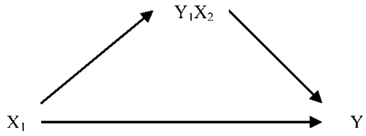

Figure 1 *Relay channel*

not be permissible because: $U =$ constant, carries no information and according to (1), it is not decodable $(X_1 \to X_2 \to Y_1)$ and hence, the relay cannot and need not decode and forward anything, whereas every AGRCDAF must decode something U of X_1.

4.1 The interpretation of the definition

1. The relay channel aims to send X_1 to Y and the receiver needs the co-operation of the relay to find X_1, otherwise the relay is useless.

2. The relay generally does not understand X_1 fully (the channel is not a degraded one in general) and only understands U of X_1, through X_2Y_1 and nothing. Mathematically $X_1 \to U \to X_2Y_1$ or necessarily (1) in the definition.

3. The relay must be better than the receiver in finding U, otherwise the relay is useless. Mathematically $U \to X_2Y_1 \to Y$, (2). In other words by $X_2 = \varnothing$, the broadcast sub-channel $(X_1 \to YY_1)$ must be a less noisy channel [19] regarding U, (sufficiently $U \to Y_1 \to Y$).

4. In useless cases of the relay, the receiver is itself better than the relay in finding U and X_1 (sufficiently $UX_1 \to Y \to Y_1X_2$ or necessarily from (6) below $X_1 \to X_2UY \to Y_1$, or sufficiently (3) accompanying (1) in the definition).

4.2 Some properties of AGRCDAF

For Markov chains on arbitrary random variables of X, W, Z and Y according to the relation

$$\begin{aligned} I(X; Z, Y|W) &= I(X; Z|W) + I(X; Y|Z, W) \\ &= I(X; Y|W) + I(X; Z|Y, W) \end{aligned} \tag{5}$$

and the non-negativity of mutual information, it is readily shown that

$$\begin{aligned} X \to W \to (Z, Y) &\Leftrightarrow \left\{ \begin{array}{c} X \to W \to Z \\ \text{and} \\ X \to (W, Z) \to Y \end{array} \right\} \\ &\Leftrightarrow \left\{ \begin{array}{c} X \to W \to Y \\ \text{and} \\ X \to (W, Y) \to Z \end{array} \right\} \end{aligned} \tag{6}$$

Lemma 1: For the AGRCDAF, we have

$$\mathrm{p}_1\colon X_1 \to X_2UY \to Y_1 \tag{7}$$

p_2: If $Y_1 = f(X_1, X_2)$ then U can be a function of X_1 and X_2

$$\mathrm{p}_3\colon I(U; Y_1|X_2) = I(X_1; Y_1|X_2) \tag{8a}$$

$$I(X_1; Y|X_2U) = I(X_1; Y|X_2Y_1) \tag{8b}$$

Proof of Lemma 1:

p_1:

$$\left. \begin{array}{l} (3) \Rightarrow Y_1 \to X_1X_2U \to Y \\ (1) \Rightarrow Y_1 \to X_2U \to X_1 \end{array} \right\} \overset{(6)}{\Rightarrow} \left\{ \begin{array}{c} (7) \\ \text{and} \\ Y_1 \to X_2U \to Y \end{array} \right\} \tag{9}$$

p_2: $(1) \Rightarrow H(Y_1 \mid X_2U) = H(Y_1 \mid X_2UX_1) = 0 \Rightarrow Y_1$ is a function of X_2 and $U \Rightarrow U$ can be a function of X_1 and X_2.

p_3: For $U = X_1$, (8a) is obvious and both sides of (8b) are zero from(2), but for $U \neq X_1$, we have

$$(4) \Rightarrow (U \to X_1X_2 \to Y_1) \tag{10}$$

then (10) and (1) $\overset{(6)}{\Rightarrow}$ (8a)

and $I(X_1; Y \mid X_2U) = H(Y|X_2U) - H(Y|X_2X_1U) \underset{(3)}{\overset{(9)}{\Rightarrow}} H(Y|X_2UY_1) - H(Y|X_2X_1UY_1) \underset{(4),(6)}{\overset{(2)}{\Rightarrow}} H(Y|X_2Y_1) - H(Y|X_1X_2Y_1) = I(X_1; Y|X_2Y_1)$.

5 The capacity of the agrcdaf

Theorem: The capacity of the AGRCDAF is given by

$$\begin{aligned} C = \sup\nolimits_{p(x_1x_2u)} \min\{&I(X_1X_2; Y), I(U; Y_1|X_2) \\ &+ I(X_1; Y|X_2U)\} \end{aligned} \tag{11}$$

where supremum is taken over all $p(x_1x_2u)$ for which (4) satisfies (1)–(3).

Proof of the theorem: Achievability: It can be directly proved using random coding and random binning but here the proof is omitted, because it can be derived from [2, theorem 7] by the substitution of $\hat{Y}_1 = \varnothing$, $V = X_2$, $U = (X_2, Q)$ and renaming Q by U, as statement R in (a_1) of Section 2 or it can be seen in [4, theorem 2.4] and in [3].

Converse: From (8a) and (8b) in Lemma 1, in both cases $U = X_1$ and $U \neq X_1$, the achievability result coincides with the max flow-min cut upper bound in [2] and the converse proof is completed. Or we can prove the converse using Fano's inequality [20] and the following inequalities the proof of which is omitted for brevity

$$I(w;\, \underline{Y}) \le \sum_{i=1}^{n} I(X_{1i}X_{2i};\, Y_i) \quad \text{and} \quad I(w;\, \underline{Y}) \le \sum_{i=1}^{n} I(U_i;\, Y_{1i}|X_{2i}) + I(X_{1i};\, Y_i|X_{2i}U_i)$$

where w is the message to be sent from the sender to the receiver and $\underline{Y}$ is the received sequence at the receiver in Fig. 1.

6 The results of the theorem

6.1 Relay channel ($U \neq \emptyset$)

Till now, for the relay channel, the capacity achievable by DAF strategy has been found in the following special cases:

1. Degraded relay channel (and Gaussian degraded relay channel) in [2].

2. Reversely degraded relay channel in [2].

3. The relay channel with full feedback in [2] and with partial feedback (receiver-relay feedback) in [21].

4. Semi-deterministic relay channel in [3].

5. The relay channel with orthogonal components in [4, 5].

Now, we prove that the capacity of all of the above special relay channels is derived from our capacity theorem. In other words, we show that our general model defined as AGRCDAF includes all of the above special models as its sub-sets.

1. If $U = X_1$ (or $U = g(X_1)$ and g is reversible), then there exists $p(x_1x_2u) = p(x_1x_2)$ such that for degraded relay channel from (4) we have $p(x_1x_2yy_1) = p(x_1x_2)p(y_1|x_1x_2)p(y|x_2x_1) \rightarrow$ (2); and (1) is obvious from $U = X_1$, then, in accordance with (2), a degraded relay channel is one AGRCDAF and (11) gives (12) in [2, theorem 1] $C = \sup_{p(x_1x_2)} \min\{I(X_1X_2;\, Y), I(X_1,\, Y_1|X_2)\}$.

2. If $U = X_2$, then there exists $p(x_1x_2u) = p(x_1x_2)$ such that for a reversely degraded relay channel from (4) we can have $p(x_1x_2yy_1) = p(x_1x_2)p(y|x_1x_2)p(y_2|x_2) \rightarrow$ (3) and (1) and also (2) is obvious from $U = X_2$; (3), (1) $\xrightarrow{(6)} (X_1 \rightarrow X_2Y \rightarrow Y_1)$ or from (7) by $U = X_2$ we have $(X_1 \rightarrow X_2Y \rightarrow Y_1)$, then, a reversely degraded relay channel is one AGRCDAF the capacity of which is obtained from (11) $C = \max_{p(x_1)} \max_{x_2} I(X_1;\, Y|x_2)$.

And, in this case, as mentioned in the definition, $U =$ constant is not permissible and the relay cannot, $(X_1 \rightarrow X_2 \rightarrow Y_1)$ or does not need (U has no information for the relay) decode anything and can send what it receives, thus, $X_2 = g(Y_1)$ or $Y_1 = f(X_2)$ (g is reversible) and (1)–(3) are obvious and (11) reduces to the capacity of reversely degraded relay channel.

Remark: The capacity of reversely degraded relay channel can also be achieved by EAF strategy.

For the reversely degraded relay channel (Fig. 1), we have

$$X_1 \rightarrow X_2Y \rightarrow Y_1 \tag{12}$$

and from [2, theorem 6], for EAF strategy, the following rate is achievable

$$R = I(X_1;\, Y\hat{Y}_1|X_2) \tag{13}$$

with the relation from the channel distribution

$$X_1Y \rightarrow X_2Y_1 \rightarrow \hat{Y}_1 \tag{14}$$

and (12), (14) $\xrightarrow{(6)}$

$$X_1 \rightarrow X_2Y \rightarrow \hat{Y}_1 \tag{15}$$

and (14), (1) $\xrightarrow{(6)}$

$$X_1 \rightarrow X_2 \rightarrow \hat{Y}_1 \tag{16}$$

then, from (13), using (15) and (16), we have

$$R = I(X_1;\, Y\hat{Y}_1|X_2) = I(X_1;\, Y|X_2) = I(X_1;\, Y|X_2\hat{Y}_1)$$

3. The capacity of the relay channel with full feedback (theorem 3 in [2]) is obtained from (11) by $U = X_1$ and $Y_1 \rightarrow (Y,\, Y_1)$

$$C_{FB} = \sup_{p(x_1x_2)} \min\{I(X_1X_2;\, Y), I(X_1;\, YY_1|X_2)\}$$

4. If $Y_1 = f\,(X_1X_2)$, then Y_1 is known at the transmitter (assuming that the transmitter knows the first symbol of X_2 and noting to the relay function) and according to the property p_2, U is also a function of X_1 and X_2 and we can put $U = Y_1$, then there exists $p(x_1x_2u) = p(x_1x_2y_1)$ such that in this case, (1)–(3) are obvious for every (4) and (11) reduces to the capacity of semi-deterministic relay channel in [3, corollary(6)]

$$C = \max_{p(x_1x_2)} \min\{I(X_1X_2;\, Y), H(Y_1|X_2) + I(X_1;\, Y|X_2Y_1)\}$$

5. If $X_1 = (X_D,\, X_R)$ and $U = X_R$, then there exists $p(x_1x_2u) = p(x_Rx_Dx_2)$ such that for orthogonal relay channel from (4) we can have

$$p(x_2x_Rx_Dyy_1) = p(x_2)p(x_R|x_2)p(x_D|x_2)p(y_1|x_2x_R)p(y|x_2x_D) \tag{17}$$

From (17), which is the definition of orthogonal relay

doi: 10.1049/iet-com.2008.0368

channel [5], we have

$$X_D \rightarrow X_2 \rightarrow X_R \tag{18}$$

$$X_D Y \rightarrow X_2 X_R \rightarrow Y_1 \tag{19}$$

$$X_R \rightarrow X_2 X_D \rightarrow Y \tag{20}$$

and (19) $\xrightarrow{(6)}$ (1) and (3), and (18) and (20) $\xrightarrow{(6)}$

$$(X_R \rightarrow X_2 \rightarrow Y) \tag{21}$$

(19) $\rightarrow$

$$(Y \rightarrow X_2 X_R \rightarrow Y_1) \tag{22}$$

(21) and (22) $\xrightarrow{(6)}$

$$(X_R \rightarrow X_2 Y_1 \rightarrow Y) \tag{23}$$

Then, a relay channel with orthogonal components in [5] becomes one AGRCDAF (or the model (17) is a sub-set of our general model) and the capacity in (11) is readily reduced to the capacity in [5] [using (20), (21)]

$$\mathcal{C} = \max_{p(x_2)p(x_R|x_2)p(x_D|x_2)} \min\{I(X_D X_2; Y), I(X_R; Y_1|X_2) + I(X_D; Y|X_2)\}$$

6.1.1 Note. Clear demarcation of novelty of our general model in relation to the other works such as [4, 5]: N_1: As mentioned before, the common part of the capacity theorem for orthogonal relay channel in [4, 5] and our capacity theorem is the achievability part. This part is a consequence of Cover and El Gamal's work [2, theorem 7] as we have written in the proof of our theorem.

N_2: The new and interesting part of our theorem is its converse and the differences are below.

a. The orthogonal relay channel in [4, 5] and other special relay channels are all sub-sets of our general model (AGRCDAF) as we have shown in the above.

b. In [4, 5] it has been proved that the common rate from [2, theorem 7] is the capacity of orthogonal relay channel only, but we have proved that this common rate is the capacity of a more general model (AGRCDAF) including orthogonal and other special relay channels.

c. The capacity theorem for orthogonal relay channel in [4, 5] is a special case of our capacity theorem and despite our theorem, it does not give the capacity of special cases such as semi-deterministic relay channel and the case $U = X_2$.

6. The feedback from the relay to the transmitter does not increase the capacity of the AGRCDAF because the achievable rate in (theorem 3, [22]) by $\hat{Y} = \varnothing$ and $\hat{V} = U$ coincides with the upper bound in (theorem 1, [22]) by $V = U$ and in accordance with (7).

6.2 Relay channel ($U = \varnothing$)

In accordance with (1) and (3), the definition considers every AGRCDAF with $U=$ constant or $\varnothing$ or X_2 as a reversely degraded relay channel the capacity of which can also be achieved by EAF strategy.

6.2.1 About the capacity of deterministic relay channel [7]: In deterministic relay channel [7], the relay finds only the bin of Y_1 by receiving Y_1 and sends it by a noiseless link having the rate R_0 to the receiver. Therefore there is not any X_2 and we have:

According to [7], the channel uses EAF strategy or hash and forward strategy and hence, our theorem does nothing to say about this channel.

6.2.2 About MSRC in [6]: If $V = 0$, we reach to the above 6.2.1. If $V \neq 0$, then, $U = \varnothing$ in our definition (the channel uses EAF strategy), the conditions (1) and (2) of AGRCDAF are satisfied, but the condition (3) is not necessarily satisfied and MSRC is not a AGRCDAF.

If in MSRC the condition (3) is satisfied, then it will be a reversely degraded relay channel using EAF strategy (this is correct, because the relay only observes the corrupted version of noise and is worse than the receiver) and our capacity theorem gives its capacity as follows ($Y \rightarrow Y$, S; $\hat{Y}_1 \rightarrow U$ and $X_2 S = N$ in [6])

$$\begin{aligned}\mathcal{C} &= I(X_1; Y\hat{Y}_1|X_2) = I(X_1; Y|X_2\hat{Y}_1) \rightarrow \mathcal{C} \\ &= I(X_1; SY|X_2\hat{Y}_1) = I(X_1; S|X_2\hat{Y}_1) + I(X_1; Y|X_2 S\hat{Y}_1) \\ &= 0 + I(X_1; Y|X_2 S\hat{Y}_1) = H(Y|X_2 S\hat{Y}_1) - H(Y|X_1 X_2 S\hat{Y}_1) \\ &= H(Y|N\hat{Y}_1) - H(Z|X_1 N\hat{Y}_1) = H(Y) - H(Z|\hat{Y}_1)\end{aligned}$$

6.3 Consistency of the theorem with previous results

We can validate our theorem by consistency of the terms in (11) with previous results (the capacity region for broadcast channel with degraded message sets [23] and multiple access channel with partially co-operating encoders (in a special case) [18]).

6.3.1 Broadcast channel with degraded message sets [23]: By $X_2 = \varnothing$, AGRCDAF (Fig. 1) is reduced to a broadcast channel with degraded message sets because:

First

$$\left.\begin{array}{ll}(3) \rightarrow & Y_1 \rightarrow X_1 U \rightarrow Y \\ (4) \rightarrow & U \rightarrow X_1 \rightarrow Y\end{array}\right\} \overset{(6)}{\Longrightarrow} \quad Y_1 \rightarrow X_1 \rightarrow Y$$

doi: 10.1049/iet-com.2008.0368

Second

$$I(U;Y_1)+I(X_1;Y|U)\overset{(2)}{\geq} I(U;Y)+I(X_1;Y|U)$$
$$=I(X_1U;Y)\overset{(4)}{=}I(X_1;Y)$$

Therefore $\min\{I(X_1;Y), I(U;Y_1)+I(X_1;Y|U)\}=I(X_1;Y)$, hence, we can establish the capacity region for broadcast channel with degraded message sets from the terms in (11) as follows (as in [23])

$$R_0 \leq I(U;Y_1), \quad R_1 \leq I(X_1;Y|U), \quad R_0+R_1 \leq I(X_1;Y)$$

6.3.2 Multiple access channel with partially co-operating encoders (in a special case) [18]: In Fig. 1, we have one-sided co-operation between the transmitters (the relay and the transmitter), then, in [18] $\mathcal{C}_{21}=0$, $\mathcal{C}_{12}=I(U;Y_1|X_2)$, $M_2=\varnothing \to R_2=0$, and from (11), the capacity region can be established the same as in [18].

7 Conclusion

We have defined AGRCDAF strategy and showed that at least the relay channels with known capacity are special cases of it. Also, we have shown that our capacity theorem is validated by its consistency with the capacity regions for broadcast channel with degraded message sets and multiple access channel with partially co-operating encoders (in a special case). We claim that the capacity of general relay channel might be described by one auxiliary random variable or more variables depending on how we define the role of the relay.

8 Acknowledgment

The authors would like to thank Prof. Rashvand and the anonymous reviewers for their helpful and invaluable comments. This work was accepted in part at the 2008 International Symposium on Information Theory and its Applications (ISITA 2008); was supported in part by Iran Telecommunications Research Center (contractT500-20613) and Iran National Science Foundation.

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