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On permutation of space-time-frequency block codings

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Abstract: Multi-input multi-output-orthogonal frequency-division multiplexing is the foundation of next generation of wireless communication systems and space-time-frequency block coding (STFBC) is considered one of the best schemes for implementing these systems. STFBCs have very complex structures and many parameters affect their performance. Studies have shown that permutation parameter plays a very important and sensitive role in the construction of STFBCs so much so that a small variation of this parameter could cause a significant change in the value of the coding advantage (CA) which alters the code's performance. In this study, first the authors explore how the permutation parameter affects STFBCs and demonstrate that existing permutation methods do not guarantee to attain the maximum possible CA. Next, they introduce a new structure for STFBCs permutation, which is the basis for the design of a modified model of one recently published STFBC. Comparison of the simulation results from the original and modified codes confirms the latter improves the performance by up to 3 dB. This is an achievement that is also verified by theoretical analysis.

1 Introduction

It is well known that multi-input multi-output (MIMO) systems offer more channel capacity and, as a result, the capability of transmission at a higher data-rate compared with single-input single-output systems. MIMO systems, by introducing diversity, play the role of modern channel coding in fading channels. To implement MIMO systems, space-time coding (STC) is a highly effective solution to handle the ruinous fading phenomenon for narrow-band communication [1–4]. Using shorter time slots to send symbols is, of course, a conventional method of achieving a high-speed data transmission, but the drawback is that this makes the channel frequency selective (FS) which can cause an extreme intersymbol interference (ISI) in addition to fading and under these circumstances MIMO coding requires a complex equaliser at the receiver. In FS channels, STC also loses the frequency diversity which is offered in the FS channels [5].

Orthogonal frequency-division multiplexing (OFDM) spreads symbols over a larger time slot using orthogonal subcarriers for modulating different symbols. This technique divides an FS channel into several frequency non-selective channels by using multicarrier modulation scheme. Multi-carrier is a proper modulation for FS channels. In addition, OFDM could be easily implemented using fast Fourier transform (FFT), where the equaliser consists of a simple division. In an FS channel with L different paths between each pair of transmit and receive antennas, there exists the potential of having L different versions of transmitted signals at the receiver. The multipath effect, which is the time domain representation of an FS channel, seems to be ruinous and causes ISI, but it could be wisely used as a source of natural diversity. To do so, MIMO-OFDM systems take advantage of both OFDM and MIMO for multipath and fading channels, respectively. Space-frequency block coding (SFBC) and space-time-frequency block coding (STFBC) are two major schemes for implementing MIMO-OFDM systems, and already many SFBCs have been put forward [6–14]. SFBCs use both frequency and spatial diversities [5, 6]. The performance criteria for SFBCs are derived in [7]. For a full-diversity SFBC, the highest available diversity is equal to LM_tM_r, where M_t and M_r are the number of transmit and receive antennas, respectively, and L is the number of the FS taps [15].

Diversity order and diversity product [coding advantage (CA)] are two parameters which are used to evaluate the performance of MIMO codes. The diversity order is somehow related to the number of independent fading coefficients that the codeword experiences. On the other hand, the diversity product, by some means, increases as the fading coefficients become more uncorrelated.

In a MIMO-OFDM system, one technique to decrease correlation of the channel frequency response that occurs because of discrete fourier transform (DFT) process at different subcarriers is to permute the active subcarriers of each block. In fact by employing a permutation method, the transmit data consisting of same symbol experiences more varying fading coefficients.

A full-rate full-diversity SFBC is proposed in [7], in which a permutation method is applied in order to maximise the code performance, where both the delay and power profiles

(DPPs) of the channel are known at the transmitter. The diversity product of this code can be decomposed into two parts; namely, the intrinsic diversity product and the extrinsic diversity product. The extrinsic diversity product depends on the applied interleaving strategy and the DPP of the channel. Then by introducing a permutation method, the extrinsic diversity product can be increased which leads to an improvement in the CA. The proposed codes in [16] use a fixed permutation distance equal to N/L for all states. This permutation method is not flexible and, therefore, the code cannot be optimised for any arbitrary DPPs to attain the maximum possible CA.

STFBCs have also the potential of taking the advantage of time diversity. The design criteria for STFBCs are provided in [15–17]. In [16], OFDM subcarriers are divided into smaller groups, but such grouping does not reduce the diversity gain. When the channel behaviour changes for different OFDM blocks, diversity gain for STFBCs is $M_\mathrm{t}M_\mathrm{r}L \times \mathrm{rank}(\boldsymbol{R}_\mathrm{T})$, where $\boldsymbol{R}_\mathrm{T}$ is temporal correlation matrix of the channel. In [18, 19], systematic designs for space time frequency coding (STFCs) are presented to achieve full-diversity STFBCs.

As in [7, 16, 19], since permutation of STFBCs can significantly change the CA value, it obviously plays a critical role as far as performance is concerned. Therefore it is important to study the structure of existing permutations and to develop more effective schemes. In this paper, we first demonstrate that the permutation methods proposed in the literature are not suitable in all the possible scenarios and then propose a novel permutation scheme which takes both temporal and frequency correlation matrices into account. The proposed method is applicable to all the existing STFBCs found in literature. To verify the claim, we will take STFBCs studied in [19], apply modification on the basis of the proposed permutation solution and examine the performance. As will be shown, the results confirm that the modified model offers up to 3 dB improvement. The rest of the paper is organised as follows.

In the next section, the basic principle of MIMO-OFDM system model is introduced. In Section 3, we review frequency and temporal correlation matrices and investigate permutation of STFBCs. This is followed by IV in which we present our permutation scheme and provide a mathematical analysis to verify the superiority of the proposed solution over the existing methods. Section 5 describes a summary of block circular delay diversity (BCDD) code in [19]. In Section 6, a modification of the BCDD codes is presented based on the proposed permutation. Section 7 presents and discusses the simulation results and conclusion of the paper is expressed in the last section.

Notations: we use capital boldface letters for matrices. Superscripts $(\cdot)^\mathrm{T}$, $(\cdot)^\mathcal{H}$ and $(\cdot)^*$ denote transpose, Hermitian and complex conjugation, respectively. $\circ$, and $\otimes$ stand for the Hadamard and the tensor products, respectively. Notation $\mathrm{diag}(a_1, a_2, \ldots, a_n)$ represents a diagonal $n \times n$ matrix whose diagonal entries are $\mathrm{diag}(a_1, a_2, \ldots, a_n)$. $\mathbb{C}$ stands for the complex field and $\boldsymbol{V}(t_1, t_2, \ldots, t_n)$ denotes a Vandermonde matrix with parameters $(t_1, t_2, \ldots, t_n)$, such that

$$\boldsymbol{V}(t_1, t_2, \ldots, t_n) = \begin{bmatrix} 1 & t_1 & t_1^2 & \cdots & t_1^{n-1} \\ 1 & t_2 & t_2^2 & \cdots & t_2^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & t_n & t_n^2 & \cdots & t_n^{n-1} \end{bmatrix} \in \mathbb{C}^{n\times n}$$

$\mathcal{J} = \sqrt{-1}$, $\lfloor\cdot\rfloor$ stands for the floor operation, 1_a indicates a $a \times a$ matrix of ones and $\boldsymbol{I}_a$ represents an $a \times a$ identity matrix.

2 System model

In this section, we describe the system model of a MIMO-OFDM system. Consider an STF-coded MIMO-OFDM system with M_t transmit antennas and M_r receive antennas and N subcarriers within K successive OFDM blocks. Channel impulse response during the kth OFDM block from the transmit antenna i to the receive antenna j is given by

$$h_{i,j}^k(\zeta) = \sum_{l=0}^{L-1} \alpha_{i,j}^k(l)\delta(\zeta - \zeta_l), \quad k = 1, 2, \ldots, K \tag{1}$$

where ζ_l's are time delays and $\alpha_{i,j}^k(l)$'s are the complex amplitude of lth path between the transmit antenna i and the receive antenna j which are modelled as zero-mean complex Gaussian random variables with variances $E\left|\alpha_{i,j}^k(l)\right|^2 = \sigma_l^2$ for normalisation purposes. The power of L paths are assumed to satisfy the condition $\sum_{l=0}^{L-1}\sigma_l^2 = 1$.

Each STF code can be formed as a $KN \times M_\mathrm{t}$ matrix

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{C}_1^\mathrm{T} & \boldsymbol{C}_2^\mathrm{T} & \cdots & \boldsymbol{C}_K^\mathrm{T} \end{bmatrix}^\mathrm{T} \tag{2}$$

where

$$\boldsymbol{C}_k = \begin{bmatrix} c_1^k(0) & c_2^k(0) & \ldots & c_{M_\mathrm{t}}^k(0) \\ c_1^k(1) & c_2^k(1) & \ldots & c_{M_\mathrm{t}}^k(1) \\ \vdots & \vdots & \ddots & \vdots \\ c_1^k(N-1) & c_2^k(N-1) & \ldots & c_{M_\mathrm{t}}^k(N-1) \end{bmatrix} \tag{3}$$

where $c_i^k(n)$ is the symbol or combination of symbols which is transmitted on the nth subcarrier by transmit antenna i in the kth OFDM block. The transmitter applies an N-point inverse FFT over each column of $\boldsymbol{C}_k$ and after adding cyclic prefix the ith column of $\boldsymbol{C}_k$ is transmitted by the transmit antenna i.

The received signal at the antenna j, in the kth OFDM block, after passing through the match filter, having the cyclic prefix removed and implementing FFT, at the nth frequency subcarrier, is given by

$$r_j^k(n) = \sum_{i=1}^{M_\mathrm{t}} c_i^k(n)H_{i,j}^k(n) + \mathcal{N}_j^k(n), \qquad n = 0, 1, \ldots, N-1 \tag{4}$$

where $H_{i,j}^k(n)$ is the channel frequency response at the nth subcarrier that is given by

$$\mathcal{F}\{h_{i,j}^k(\xi)\} = H_{i,j}^k(f)|_{f=n\Delta f} \triangleq H_{i,j}^k(n) = \sum_{l=0}^{L-1}\alpha_{i,j}^k(l)\mathrm{e}^{-j2\pi n\delta f\xi_l} \tag{5}$$

The symbol $\mathcal{F}$ represents the Fourier transform, $\Delta f = 1/T_d = BW/N$, T_d is OFDM symbol period and BW is total bandwidth. $\mathcal{N}_j^k(n)$ denotes the zero-mean additive white complex Gaussian noise with unit variance corresponding to the nth frequency subcarrier at the receive antenna j and the kth OFDM symbol duration.

3 Design criteria and frequency and temporal correlation matrices

The CA of an SFBC depends on both the code structure and channel characteristics. It has already been proved that in an L-ray channel an STFBC could achieve the diversity order of rLM_t, where r is the rank of temporal correlation matrix. Hence, to achieve maximum available diversity at least each of LM_t rows of the matrix $\boldsymbol{C}_k$ must be constructed together and they must contain information about the same symbols. To minimise the complexity of the receiver while attaining maximum diversity, most existing SFBCs are designed by using sub-blocks of LM_t rows of the code which are joined together to construct a certain type of SFBC. Therefore the construction of any STFBC could be described based on the design of the sub-blocks of size $\Gamma M_t \times M_t$ matrices $\boldsymbol{G}_k^{\mathrm{p}}$, for $p = 1, 2, \ldots, N/\Gamma\ M_t$, where $1 \le \Gamma \le \mathrm{L}$, with N being a multiple of ΓM_t. Note that this condition does not cause any concern since zero padding or the use of smaller code blocks could resolve this problem.

Let us assume each codeword of an STFBC can be written as follows

$$\boldsymbol{C}_k = \begin{bmatrix} \boldsymbol{G}_k^1 \\ \boldsymbol{G}_k^2 \\ \vdots \\ \boldsymbol{G}_k^{N_c/\mathrm{L}M_t} \end{bmatrix} \in \mathbb{C}^{N \times M_t} \tag{6}$$

as the primary codeword $\boldsymbol{G}_\mathrm{p}$ defined as

$$\boldsymbol{G}_\mathrm{p} = \begin{bmatrix} \boldsymbol{G}_1^{\mathrm{p}} \\ \boldsymbol{G}_2^{\mathrm{p}} \\ \vdots \\ \boldsymbol{G}_K^{\mathrm{p}} \end{bmatrix} \in \mathbb{C}^{\boldsymbol{K}\Gamma M_t \times M_t} \tag{7}$$

Then, design and analysis of the STFBC would split into its $K\Gamma\ M_t \times M_t$ sub-matrices $\boldsymbol{G}_\mathrm{p}$'s.

The pairwise error probability between two distinct codewords $\boldsymbol{C}$ and $\tilde{\boldsymbol{C}}$ is shown to be upper bounded as [15]

$$P\big(\boldsymbol{C} \to \tilde{\boldsymbol{C}}\big) \le \binom{2\nu M_r - 1}{\nu M_r} \left(\prod_{i=1}^{\nu} \lambda_i\right)^{-M_r} \left(\frac{\rho}{M_t}\right)^{-\nu M_r} \tag{8}$$

where λ_i's are the non-zero eigenvalues, ν is the minimum rank of $\boldsymbol{\Delta}\circ\boldsymbol{R}$, $\boldsymbol{\Delta} \triangleq \big(\boldsymbol{C} - \tilde{\boldsymbol{C}}\big)\big(\boldsymbol{C} - \tilde{\boldsymbol{C}}\big)^{\mathrm{H}}$, $\boldsymbol{R} \triangleq \boldsymbol{R}_\mathrm{T} \otimes \boldsymbol{R}_\mathrm{F}$ with $\boldsymbol{R}_\mathrm{T}$ and $\boldsymbol{R}_\mathrm{F}$ denoting the temporal and frequency correlation matrices, respectively, which will be discussed in detail at the end of this section.

Now, we could simply use the inequality relationship (8) for $\boldsymbol{G}_\mathrm{p}$ as

$$P\big(\boldsymbol{G} \to \tilde{\boldsymbol{G}}\big) \le \binom{2\varsigma M_r - 1}{\varsigma M_r} \left(\prod_{i=1}^{\varsigma} \gamma_i\right)^{-M_r} \left(\frac{\rho}{M_t}\right)^{-\varsigma M_r} \tag{9}$$

where γ_i's are the non-zero eigenvalues of $\hat{\boldsymbol{\Delta}} \circ \hat{\boldsymbol{R}}$, with $\hat{\boldsymbol{\Delta}} \triangleq \big(\boldsymbol{G} - \tilde{\boldsymbol{G}}\big)\big(\boldsymbol{G} - \tilde{\boldsymbol{G}}\big)^{\mathrm{H}}$, and $\hat{\boldsymbol{R}}$ being a principal sub-matrix of $\boldsymbol{R}$ with the same indexing which $\hat{\boldsymbol{\Delta}}$ lies in the matrix $\boldsymbol{\Delta}$. Matrices $\boldsymbol{G}$ and $\hat{\boldsymbol{G}}$ are two distinct primary codewords of size $K\Gamma M_t \times M_t$.

The diversity order of STFBC is equal to ν and the minimum value of $\prod_{i=1}^{\nu} \lambda_i$ is known as the CA.

Using a similar structure for G_p's and taking two distinct codewords $\boldsymbol{C}$ and $\tilde{\boldsymbol{C}}$, there is at least one index $p_0(1 \le p_0 \le N/LM_t)$ such that $\boldsymbol{G}_{\mathrm{p}_0}$ and $\tilde{\boldsymbol{G}}_{\mathrm{p}_0}$ are different. We may further assume that $\boldsymbol{G}_\mathrm{p} = \tilde{\boldsymbol{G}}_\mathrm{p}$ for any $p \ne p_0$ since the rank of $\boldsymbol{\Delta}\circ\boldsymbol{R}$ does not decrease if $\boldsymbol{G}_\mathrm{p} \ne \tilde{\boldsymbol{G}}_\mathrm{p}$ for some $p \ne p_0$. Thus, the rank and the CA of an STFBC are equal to the primary codeword described in (9), that is, $\nu = \varsigma$ and $\min\big(\prod_{i=1}^{\nu} \lambda_i\big) = \min\big(\prod_{i=1}^{\varsigma} \gamma_i\big)$.

For full-diversity STFBCs with $\Gamma = L$, the matrix $\hat{\boldsymbol{\Delta}} \circ \hat{\boldsymbol{R}}$ is full-rank. Hence, all the eigenvalues of $\hat{\boldsymbol{\Delta}} \circ \hat{\boldsymbol{R}}$ are non-zero and we could calculate the CA by utilising the following determinant

$$CA = \min\left(\prod_{i=1}^{\nu} \gamma_i\right) = \min\big(\det\big(\hat{\boldsymbol{\Delta}} \circ \hat{\boldsymbol{R}}\big)\big), \quad \nu = \mathrm{L}M_t \tag{10}$$

Clearly, the CA of STFBCs is related to $\boldsymbol{R}$ which represents correlation of the channel impulse response, the DPPs and correlation of channel coefficients of different time slots. As diversity means in MIMO systems, signals must experience independent fading coefficients and when these coefficients are not linearly independent, the more uncorrelated they are the greater will be the CA achieved by the coding scheme.

In a space-time-frequency coded MIMO-OFDM system with L-ray channel between the transmit and the receive antennas in a quasi-static channel, (where the channel impulse response does not change while the entire codeword of an STFBC is being sent) there are only LM_t linearly independent fading coefficients at each of the receive antennas over the duration of $K\Gamma M_t^2$ symbols transmission. Since under this condition it is not possible to obtain higher diversity, which is required for achieving a better performance, we need to maximise the CA by using the most uncorrelated coefficients for the $K\Gamma M_t^2$ symbols.

Interleaving the rows of the STFBC codeword is a reasonable answer to this problem. From a survey of recent literature, the method used for interleaving STFBCs is to permute the rows of each OFDM block. For SFBCs this is the only possible way out, but for STFBCs there are other options. In this section, we first show how existing interleaving methods work and then propose a simple scheme, which brings about a significant improvement in STFBCs.

In a MIMO-OFDM system, correlation of the channel frequency response occurs at different subcarriers because of the DFT procedure. By permuting active subcarriers corresponding to each block, the transmit data consisting of the same symbol experience fading coefficients whose values are very different. This permutation method improves the CA of SFBCs.

The channel frequency response vector between the ith transmit antenna and the jth receive antenna at kth OFDM block will be denoted by [7]

$$H_{i,j}^k = \begin{bmatrix} H_{i,j}^k(0) & H_{i,j}^k(1) & \cdots & H_{i,j}^k(N-1) \end{bmatrix}^{\mathrm{T}} \tag{11}$$

$H_{i,j}^k$ can be decomposed as $H_{i,j}^k = W A_{i,j}^k$, with

$$W = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \mathcal{W}^{\zeta_0} & \mathcal{W}^{\zeta_1} & \cdots & \mathcal{W}^{\zeta_{L-1}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{W}^{(N-1)\zeta_0} & \mathcal{W}^{(N-1)\zeta_1} & \cdots & \mathcal{W}^{(N-1)\zeta_{L-1}} \end{bmatrix}_{N \times L} \tag{12}$$

where $\mathcal{W} = \mathrm{e}^{-\mathrm{j}2\pi\Delta f}$, if all of the L delay paths fall at the sampling instances of the receiver, W becomes part of the DFT matrix, which is unitary, also $A_{i,j}^k = \left[\alpha_{i,j}^k(0)\ \alpha_{i,j}^k(1) \cdots \alpha_{i,j}^k(L-1)\right]^{\mathrm{T}}$ which is related to the power distribution of the channel impulse response [7].

The frequency correlation matrix between the ith transmit antenna and the jth receive antenna can be calculated as

$$\begin{aligned} R_{i,j} &= E\left\{H_{i,j}^k H_{i,j}^{k\mathcal{H}}\right\} = W\left\{A_{i,j}^k A_{i,j}^{k\mathcal{H}}\right\} W^{\mathcal{H}} \\ &= W \mathrm{diag}\left(\sigma_0^2,\ \sigma_1^2,\ \ldots,\ \sigma_{L-1}^2\right) W^{\mathcal{H}} = \boldsymbol{R}_{\mathrm{F}} \end{aligned} \tag{13}$$

According to (9) and (10), in order to evaluate the performance of an SFBC using the CA, we could consider $\boldsymbol{G}_{\mathrm{p}_0}$ matrix without losing the generality. Let us take the case where $p_0 = 1$. Now, to illustrate graphically the effect of the permutation on SFBCs, we set $K = 1$. Fig. 1 displays matrix $\hat{\boldsymbol{R}}$, where a space-frequency coded system with six transmit antennas and a 2-ray equal power FS channel with 5 µs delay spread and $N = 128$ subcarriers is used. The horizontal axis indicates the indices of the rows and the columns of matrix $\hat{\boldsymbol{R}}$ and the vertical axis indicates the absolute values of $\hat{\boldsymbol{R}}$. As can be seen, the fading coefficients are highly correlated which, as argued above, increases the CA.

Now, we can show that applying a simple permutation to the coding system makes the subcarriers corresponding to each block of the code fall apart. For example, in the scenario under investigation, it is clear that $\hat{\boldsymbol{R}}$ lies within the index set $\{1, 2, \ldots, 12\}$, but after carrying out a simple permutation the index set changes to $\{1, 6, 12, \ldots, 67\}$ which in turn changes $\hat{\boldsymbol{R}}$, Fig. 2 depicts the effect of this process.

Temporal correlation matrix represents the similarity between the fading coefficients of different OFDM blocks. In an extreme case of quasi-static channel, fading coefficient values of each subcarrier stays constant for K OFDM blocks. This means that under the circumstances where the indexing of all K OFDM blocks is identical symbols that contain the same data experience similar fading.

Temporal correlation in mth delay is

$$r_K(m) = E\left\{\alpha_{i,j}^k(l)\alpha_{i,j}^{k+m}(l)^*\right\} \tag{14}$$

Therefore temporal correlation matrix of size $K \times K$ can be written as

$$R_{\mathrm{T}}(k,\ p) = v(k-p) \tag{15}$$

where $v(k-p) = E\left\{\alpha_{i,j}^k(l)\alpha_{i,j}^{\mathrm{p}}(l)^*\right\}$. Let us suppose $r_{\mathrm{T}} = \mathrm{rank}(R_{\mathrm{T}})$ with $r_{\mathrm{T}} \le K$. Then, we have: $R_{\mathrm{T}} = V_{R_{\mathrm{T}}} \Lambda_{R_{\mathrm{T}}} V_{R_{\mathrm{T}}}^{\mathcal{H}}$ which is derived from eigenvalue decomposition. Here, $\Lambda_{R_{\mathrm{T}}}$ contains the r_{T} non-zero eigenvalues and $\boldsymbol{V}_{R_{\mathrm{T}}}$ is the corresponding $K \times r_{\mathrm{T}}$ eigenvector matrix. We assume $\alpha_{i,j}^k(l) = \varepsilon\alpha_{i,j}^{k-1}(l) + \vartheta_{i,j}^k(l)$, where $\left\{\vartheta_{i,j}^k(l)\right\}$ are i.i.d. zero-mean complex Gaussian with variance $\sigma_l^2\left(1-\varepsilon^2\right)$, $0 \le \varepsilon \le 1$ [15].

4 Proposed permutation scheme

To the best of authors' knowledge, permutation schemes published to date for STFBCs offer more or less the same structure, that is, the same permutation is applied to each OFDM block. In all these cases, one parameter is defined that represents the distance between each two rows of each block of the codeword after permutation has been performed.

Taking a different approach from those in literature, we propose another vector of parameters, namely $K_{\mathrm{s}} = \{K_{\mathrm{s}1}, K_{\mathrm{s}2}, \ldots, K_{\mathrm{s}K}\}$ where $K_{s_i} \in \{1, 2, \ldots, N\}$, which represents the index of the first subcarrier assigned to the sub-block $\boldsymbol{G}_1^1 - \boldsymbol{G}_K^1$ for each of K OFDM blocks. Using a circular shift, this indexing method would automatically apply to all the sub-blocks $\boldsymbol{G}_k^p$ for $p = 1, 2, \ldots, N/\Gamma\ M_{\mathrm{t}}$ and $k = 1, 2, \ldots, K$. Using the CA of the STFBC, optimum values for

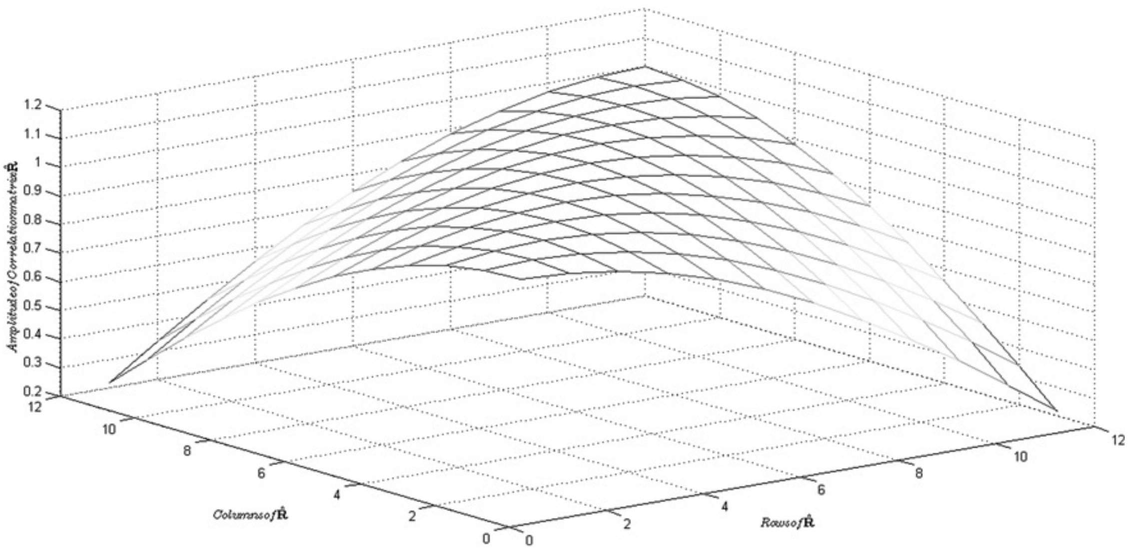

Fig. 1 *Frequency correlation between subcarriers corresponding to sub-block $\boldsymbol{G}_l$ without permutation*

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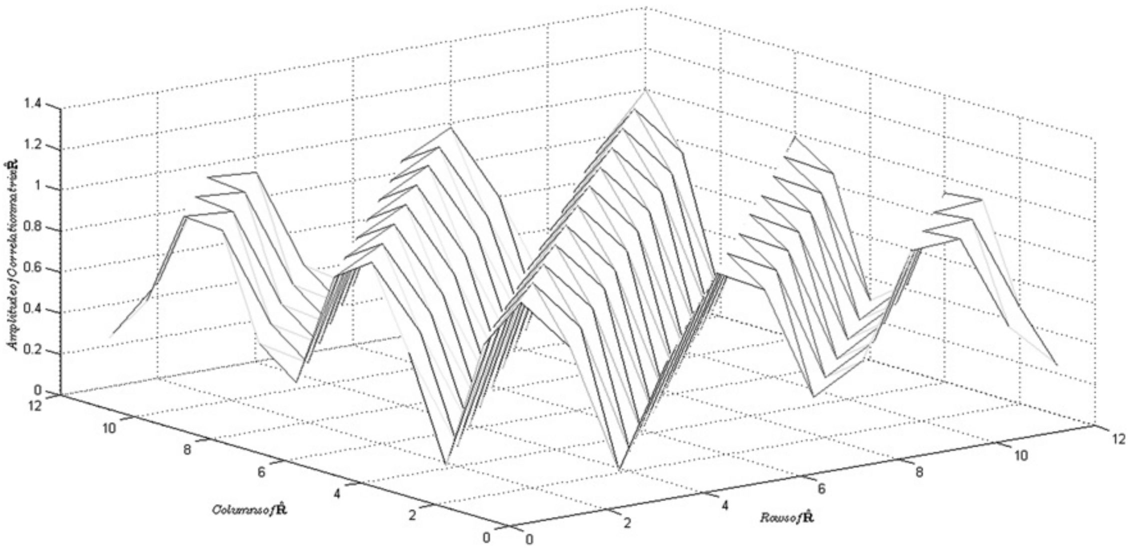

Fig. 2 *Frequency correlation between subcarriers corresponding to sub-block* $\boldsymbol{G}_l$ *with permutation*

K_{s_i} $i = 1, 2, \ldots, K$, namely, $\widehat{K_{s_i}}$ can be calculated and from the result the set $\widehat{K}_s = \left\{\widehat{K_{s_1}},\ \widehat{K_{s_2}},\ \ldots,\ \widehat{K_{s_K}}\right\}$ is obtained which is then utilised to construct a new STFBC codeword.

Since the structure of permutation is completely dependent on the structure of STFBC, it is not possible to design a general purpose algorithm. Nevertheless, in this section we illustrate that how the proposed permutation scheme can enhance the performance of STFBCs.

As discussed earlier, the diversity order and the CA are the only two criteria which give an indication of the STFBCs performance. It can be readily demonstrated that permutation parameter of STFBCs does not have any effect on their diversity order [7, 19]. A simple explanation is that since permutation can be defined as multiplication of a full-rank permutation matrix by the codeword, therefore, rank of $\hat{\boldsymbol{\Delta}} \circ \hat{\boldsymbol{R}}$ does not change. The CA, however, is directly related to the permutation and this relationship stems from the AC's dependency on the non-zero eigenvalues of $\hat{\boldsymbol{\Delta}} \circ \hat{\boldsymbol{R}}$, where $\hat{\boldsymbol{R}}$ is a submartix of $\boldsymbol{R}$ with indices which are determined by the code's permutation parameters.

Now, given the proposed permutation and also from (6) to (10), the CA of STFBCs with proposed permutation can be expressed as

$$CA_{\mathrm{NP}}\left(\widehat{K}_s\right) = \min\left(\prod_{i=1}^{v} \xi_i\right) \tag{16}$$

and

$$\widehat{K}_s = \operatorname{argmax}\ CA_{\mathrm{NP}}\left(K_s\right) \tag{17}$$

where ξ_i's are the non-zero eigenvalues of $\left(\hat{\boldsymbol{\Delta}} \circ \hat{\boldsymbol{R}}\right)_{K_s}$, the superscript (.) K_s indicates that the first subcarrier of sub-blocks $\boldsymbol{G}_1^1$ to $\boldsymbol{G}_K^1$ are $\widehat{K_{s_1}}$–$\widehat{K_{s_K}}$, respectively, and $\boldsymbol{G}_1^p$–$\boldsymbol{G}_K^p$ are shifted down with the same values. The indices which exceed the number of the subcarriers can then be mapped to the remainder of their division by N.

It is obvious that selecting the Ks as a set of zeros with cardinality of K, that is, $K_s = 0 = \{0,\ 0, \ldots,\ 0\}$ is as if the proposed permutation scheme is not being used and from (16) we can just calculate the CA of STFBC using its conventional structure, that is,. $CA_{\mathrm{NP}}(0)$.

Now, we investigate how the proposed permutation improves the CA which in turn leads to better performance.

Theorem: denoting the CA in the proposed permutation as $CA_{\mathrm{NP}}\left(\widehat{K}_s\right)$ and that in conventional STFBCs as $CA_{\mathrm{NP}}(0)$, the following inequality is satisfied

$$CA_{\mathrm{NP}}\left(\widehat{K}_s\right) \geq CA_{\mathrm{NP}}(0) \tag{18}$$

Proof: the proof for this theorem is straight forward, if

$$\widehat{K}_s = \operatorname{argmax}\ CA_{\mathrm{NP}}\left(K_s\right) = 0 \tag{19}$$

then $CA_{\mathrm{NP}}\left(\widehat{K}_s\right) = CA_{\mathrm{NP}}\left(0\right)$, and if for any other set K_s^0

$$\widehat{K}_s = \operatorname{argmax}\ CA_{\mathrm{NP}}\left(K_s\right) = K_s^0 \tag{20}$$

then $CA_{\mathrm{NP}}\left(\widehat{K}_s\right) > CA_{\mathrm{NP}}\left(0\right)$. □

To illustrate how these new parameters must be added to the structure of an STFBC, we apply the proposed algorithm to [19] which is the latest STFBC in literature and then examine the modified coding solution to see its effect on performance. In fact, in Section 7, it will be shown that an improvement of up to 3 dB can be achieved as a result of this modification. Using the same guideline, the new permutation method can also be applied to other STFBCs and it is reasonable to expect to yield improved performances. In the next section, a summary of STFBCs developed in [19] is presented.

5 Summary of BCDD codes

In [19], a systematic construction for linear transform-based STF (LT-STF) codes with high CA is presented. LT-STF codes are the generalised form of codes in [7, 15, 20, 21].

The LT-STF code, $\boldsymbol{C}$, is a $KN \times M_t$ matrix that is generated as

$$\boldsymbol{C} = [\ \boldsymbol{G}_1\underline{x}, \quad \boldsymbol{G}_2\underline{x}, \quad \cdots, \quad \boldsymbol{G}_{M_t}\underline{x}] \tag{21}$$

where

$$\underline{x}(\mathcal{M}_i) = V_i\,\underline{s}(\mathcal{M}_i), \quad i \in [1, N_{\text{part}}] \tag{22}$$

where $\underline{\boldsymbol{s}}$ is a vector of KN data symbols. The elements of $\underline{x}(\mathcal{M}_i)$ are combination of existing symbol at $\underline{s}(\mathcal{M}_i)$, and $\{\mathcal{M}_i\}_{i=1}^{N_{\text{part}}}$ denotes any partition of $\{1, 2, \ldots, KN\}$, $\{\boldsymbol{G}_i\}_{i=1}^{M_t}$ are mapping matrices of size $KN \times KN$ and constellation precoder matrices V_i are $|\mathcal{M}_i| \times |\mathcal{M}_i|$ that are used to achieve diversity and raise the CA. In [19], for V_i the Vandermonde structure is chosen.

There are two possible scenarios at the transmitter of a MIMO-OFDM system; namely, when there is partial channel state information (PCSI) (channel DPP and temporal correlation) at the transmitter, and when channel DPP and temporal correlation are unknown (no CSI). Therefore, $\{\mathcal{M}_i\}_{i=1}^{N_{\text{part}}}$ and $\{\boldsymbol{G}_i\}_{i=1}^{M_t}$ are designed for the two cases of partially known and of unknown CSI.

In [19], two families of STFBCs were proposed based on the presence or absence of channel characteristics which, for simplicity, we refer to them as BCDD as in [19]. Next in this section, a brief description of BCDD is provided.

5.1 No CSI

In the absence of CSI, $\{\mathcal{M}_i\}_{i=1}^{N_{\text{part}}}$ and $\{\boldsymbol{G}_i\}_{i=1}^{M_t}$ are designed as follows:

For $i \in [1, N_{\text{part}}]$, and $j \in [1, M_t]$, $\{\mathcal{K}_{i,j}\}_{i,j}$ is chosen as a partition of $[1, N]$. $\mathcal{K}_{i,j} = \{l_{i,j} + n\gamma_{\text{dpi}}\}_{n=0}^{L_{\text{eq}}-1}$, in which γ_{dpi} is the absolute difference between any two consecutive elements of $\mathcal{K}_{i,j}$ and $l_{i,j} = k\gamma_{\text{dpi}}L + pM_t + j$ with $i - 1 = k\gamma_{\text{dpi}}/M_t + p$, $p \in [0, \gamma_{\text{dpi}}/M_t - 1]$, $k \in [0, N/(\gamma_{dpi}L) - 1]$, let $\mathcal{B}_{i,j} = \cup_{l=1}^{K}\{(l-1)N + \mathcal{K}_{i,j}\}$ and obtain $\mathcal{M}_i = \cup_{j=1}^{M_t}\mathcal{B}_{i,j}$ ($N_{\text{part}} = N/(LM_t)$, $|\mathcal{M}_i| = KLM_t$).

We have $\boldsymbol{G}_m = \boldsymbol{I}_k \otimes \overline{\boldsymbol{G}_m}$ so that $\mathcal{K}_{i,j}$ result in

$$\begin{aligned}\overline{\boldsymbol{G}_m} &= \beta\boldsymbol{I}_{N/M_t} \otimes \text{diag}\left\{e^{[\pm j2\pi l(m-1)/M_t]}\right\}_{l=0}^{M_t-1} \\ &= \beta\boldsymbol{I}_{N/M_t} \otimes \text{diag}\left\{e^{[\pm j2\pi l/N(m-1)N/M_t]}\right\}_{l=0}^{M_t-1}\end{aligned} \tag{23}$$

5.2 Partial channel state information

When CSI is known at the transmitter, K is chosen so that the resulting R_T is full-rank. In this case $\mathcal{M}_i$ is designed as follows: $\mathcal{M}_i = \cup_{l=1}^{K}\{(l-1)N + \mathcal{J}_i\}$, $\mathcal{J}_i = \{\beta_i L_{\text{eq}}M_t + k\gamma_{\text{op}} + (i - \beta_i)\}_{k=0}^{L_{\text{eq}}M_t-1}$, $\beta_i = \lfloor(i-1)/\gamma_{\text{op}}\rfloor\gamma_{\text{op}}.G_m(\mathcal{M}_i)$ is designed as $\boldsymbol{G}_1(\mathcal{M}_1) = \boldsymbol{I}_{KL_{\text{eq}}M_t}$, and

$$\begin{aligned}\boldsymbol{G}_{\text{m}}(\mathcal{M}_1) = &\ \text{diag}\left(\{e^{-jw_{m,k}}\}_{k=1}^{K}\right) \\ &\otimes \text{diag}\left(\{e^{-j2\pi n\gamma_{op}\theta_m/N}\}_{n=0}^{L_{\text{eq}}M_t-1}\right),\end{aligned} \tag{24}$$

for $m > 1$, $w_{m,k} = 2\pi(m-1)(k-1)/K$, and θ_m is chosen so that $\det\left[\boldsymbol{S}_1\left(\boldsymbol{I}_{M_t} \otimes \Lambda_{R_T} \otimes \Delta\right)\boldsymbol{S}_1^*\right]$, as a part of CA obtained in [19], and consequently CA is the maximum. $\boldsymbol{S} = [\boldsymbol{G}_1\boldsymbol{U}, \ \boldsymbol{G}_2\boldsymbol{U}, \ \ldots, \ \boldsymbol{G}_{M_t}\boldsymbol{U}\ ]$ so that $\boldsymbol{S}(\mathcal{B}_{i,j})[S(\mathcal{B}_{i,l})]^* = 0$, $j, l \in [1, M_t]$, $j \neq l$, $\boldsymbol{S}_i = \boldsymbol{S}(\mathcal{M}_i)$, $\boldsymbol{S}_i = [\boldsymbol{G}_1(\mathcal{M}_i), \boldsymbol{G}_2(\mathcal{M}_i), \ \ldots, \ \boldsymbol{G}_{M_t}(\mathcal{M}_i)] \times \left(\boldsymbol{I}_{M_t} \otimes \boldsymbol{U}(\mathcal{M}_i)\right)$, where $\boldsymbol{U} = \boldsymbol{V}_{R_T} \otimes \boldsymbol{W}$ and $\boldsymbol{W}$ is a $N \times L$ vandermonde matrix, that $\boldsymbol{W}(p, q) = e^{-j2\pi(p-1)\tau_{q-1}/T}$ and $\Delta = \text{diag}\left(\{\delta_l^2\}_{l=0}^{L-1}\right)$.

6 Modification of BCDD codes

The coding scheme described in the previous section is capable of achieving full diversity of time, space and frequency. To obtain a high CA, one must take care with the construction of these LT-STF parts; namely, constellation precoder matrices $\{\boldsymbol{V}_i\}_i$, mapping matrix $\{\boldsymbol{G}_i\}_{i=1}^{M_t}$ and indexing set $\{\mathcal{M}_i\}_{i=1}^{N_{\text{part}}}$.

The precoder design for fading channels is studied in [22–24] and Vandermonde matrix, $V(\theta_1, \theta_2, \ \ldots, \theta_{\text{p}})$ is suggested to have a proper performance in cases of SF and STF codes [7].

The indexing set $\mathcal{M}_i$, unlike the other parts, is exclusive to OFDM modulated MIMO coded systems and mainly depends on the characteristics of the channel. To the best of authors' knowledge, $\mathcal{M}_i$ which is commonly employed for STFBCs utilises the same structure for different OFDM blocks, that is, the permutation applied to each OFDM block is the same. In other words, $\mathcal{M}_i$ which have been proposed in [19] permutes the same rows of the code in each OFDM block.

Temporal correlation matrix can represent the similarity between the fading coefficients of different OFDM blocks. In an extreme case of a static channel, fading coefficient of each subcarrier stays constant for K OFDM blocks. This means that if the indexing of all K OFDM blocks is identical, the symbols containing the same data experience similar fading coefficients, which reduces the CA of STFBC. Now, from latest literature on permutation, it is obvious that these schemes do not decrease correlation between different OFDM blocks, but only between different subcarriers in each OFDM block. Therefore, what is proposed in this study is a new $\mathcal{M}_i$ construction approach for STFBC that to a great extent overcomes this challenge. In this study, however, an innovative $\mathcal{M}_i$ construction approach for STFBC is proposed that aims to overcome the challenge. By adopting this model, in addition to frequency correlation, correlation between different OFDM blocks (temporal correlation) also decreases.

To pursue our goal, we focus on BCDD and use an enhanced structure for $\tilde{\mathcal{M}}_i$ to improve performance. In what follows, we describe the design of a modified BCDD (M-BCDD), and define indexing set for both PCSI and no CSI.

6.1 No CSI

Using the definitions of last section, we construct $\{\mathcal{K}_{i,j}\}_{i,j}$ in the same manner. $\{\mathcal{K}_{i,j}\}_{i,j}$ is chosen as a partition of $[1, N]$

$$\mathcal{K}_{i,j} = \left\{l_{i,j} + n\gamma_{\text{dpi}}\right\}_{n=0}^{L_{\text{eq}}-1} \tag{25}$$

and $l_{i,j} = k\gamma_{\text{dpi}}L_{\text{eq}} + pM_t + j$, $i - 1 = k\gamma_{\text{dpi}}/M_t + p$, where

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$p \in [0,\ \gamma_{\mathrm{dpi}}/M_t - 1],\ 0 \in [0,\ N/((\gamma\_\mathrm{dpi}\ \mathrm{L\_eq})) - 1]$ for $i \in [1, N_{\mathrm{part}}], j \in [1, M_t]$.

Then $\tilde{\mathcal{B}}_{i,j}$ is expressed as

$$\tilde{\mathcal{B}}_{i,j} = \begin{cases} \cup_{l=1}^{K}\left\{(l-2)N + K_{s_l} + \mathcal{K}_{i,j}\right\}, \\ \quad (l-1)N + K_{s_l} + \mathcal{K}_{i,j} > l \times N \\ \cup_{l=1}^{K}\left\{(l-1)N + K_{s_l} + \mathcal{K}_{i,j}\right\}, \\ \quad (l-1)N + K_{s_l} + \mathcal{K}_{i,j} < l \times N \end{cases} \tag{26}$$

where $\boldsymbol{K}_{\mathbf{sl}}$ is a vector of K parameters which represents the effect of temporal correlation matrix on indexing set. $K_s = \{K_{s_1}, K_{s_2}, \ldots, K_{s_K}\}$, $0 \le K_{s_1} \le N$.. $\tilde{\mathcal{M}}_i = \cup_{j=1}^{M_t} \tilde{\mathcal{B}}_{i,j}$ $(N_{\mathrm{part}} = N/(L_{\mathrm{eq}}M_t)$, $|\mathcal{M}_i| = KL_{\mathrm{eq}}M_t)$.

6.2 Partial channel state information

With same $\mathcal{J}_i$ we have

$$\mathcal{J}_i = \left\{\beta_i L_{\mathrm{eq}} M_t + k\gamma_{\mathrm{op}} + (i - \beta_i)\right\}_{k=0}^{L_{\mathrm{eq}}M_t - 1}, \quad \beta_i = (i-1)/\gamma_{\mathrm{op}}\gamma_{\mathrm{op}}, \quad i \in [1,\ N_{\mathrm{part}}] \tag{27}$$

Then, the proposed set for $\tilde{\mathcal{M}}_i$ is given by

$$\tilde{\mathcal{M}}_i = \begin{cases} \cup_{l=1}^{K}\left\{(l-2)N + K_{s_l} + \mathcal{J}_i\right\}, \\ \quad (l-1)N + K_{s_l} + \mathcal{J}_i > l \times N \\ \cup_{l=1}^{K}\left\{(l-1)N + K_{s_l} + \mathcal{J}_i\right\}, \\ \quad (l-1)N + K_{s_l} + \mathcal{J}_i < l \times N \end{cases} \tag{28}$$

and $K_s = \{K_{s_1}, K_{s_2}, \ldots, K_{s_K}\}$, $0 \le K_{s_1} \le N$. It is worth mentioning that the first entry of K_s, that is, K_{s_1} can be set to zeros because the relative distance of K_{s_1} is important.

7 Simulation results

In this section, we provide the simulation results of BCDD and M-BCDD for two scenarios of having channel characteristics (PSCI) and when there is no knowledge about the statistics of the channel at the transmitter (no CSI). In both scenarios, total bandwidth of 1 MHz and BPSK constellation are used to perform simulation and evaluate the results.

7.1 No CSI

Considering a MIMO-OFDM system with two transmit antennas and one receive antenna, 128 frequency tones and an equal power 2-ray quasi-static channel with $K = 2$. BCDD was constructed as in Section 5.1 and M-BCDD had the structure introduced in Section 6.1.

Under these conditions, simulations for channels with 5 and 1 μs delay spreads were carried out. Fig. 3 plots BCDD and M-BCDD for 5 μs delay spread with $K_s = \{0, 33\}$ for M-BCDD. Fig. 4 displays the results for 1 μs delay spread channel in which $K_s = \{0, 50\}$ for M-BCDD.

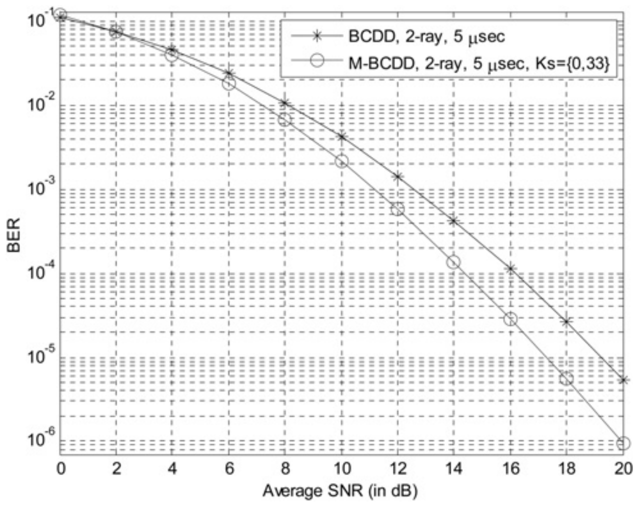

Fig. 3 *BER against SNR for the 2-ray equal power channel model with delay spread 5 μs, No CSI*

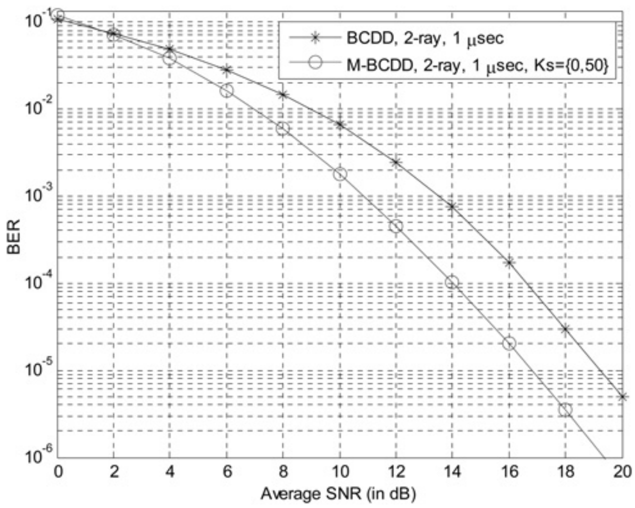

Fig. 4 *BER against SNR for the 2-ray equal power channel model with delay spread 1 μs, No CSI*

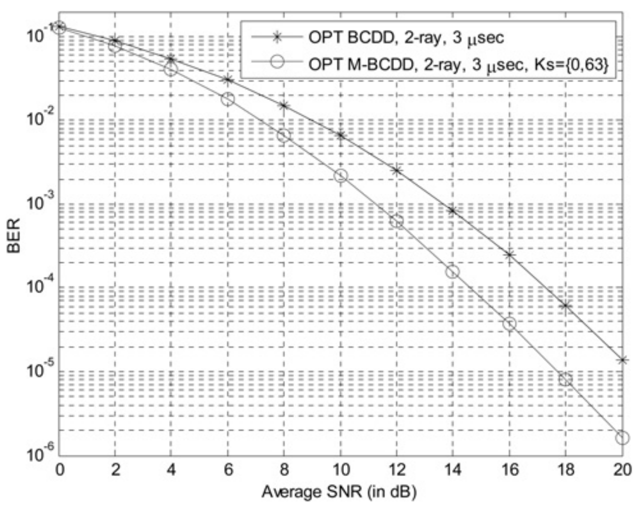

Fig. 5 *BER against SNR for the 2-ray equal power channel model with delay spread [0, 3] μs, PCSI*

7.2 Partial channel state information

Here, simulations were run for a 2-ray channel and a 4-ray channel under the following set up.

Considering a quasi-static channel with delay profile: $\underline{\tau} = [0,\ 3]\,\mu s$ and power profile: $\Delta = \text{diag}(0.5,\ 0.5)$, using two antennas at the transmitter and, one antenna at the receiver, $N = 128$, $T_d = 128\,\mu s$, $L = L_{eq} = 2$ and $K = 2$. Optimisation for BCDD in this case results in $\theta = 6$ and $\gamma_{op} = 32$. Using the M-BCDD structure proposed in Section 6.2 for optimisation, we obtained $\theta = 14$, $\gamma_{op} = 32$, $K_{s_1} = 0$ and $K_{s_2} = 63$. In Fig. 5, the results for both BCDD and M-BCDD are exhibited.

To simulate the 4-ray channel, we opted for the one used in [19] for a nearly quasi-static situation where temporal correlation $\epsilon = 0.95$, delays at $\underline{\tau} = [0,\ 5,\ 10,\ 15]\,\mu s$, and power profile $\Delta = \text{diag}(0.25,\ 0.25,\ 0.25,\ 0.25)$, with two antennas at the transmitter and one antenna at the receiver, $N = 64$, $T_d = 64\,\mu s$, $L = L_{eq} = 4$ and $K = 2$. Optimisation for BCDD in this case resulted in $\theta = 20$ and $\gamma_{op} = 8$ and using the structure proposed in Section 6.2 for optimisation M-BCDD, we obtained $\theta = 28$, $\gamma_{op} = 8$, $K_{s_1} = 0$ and $K_{s_2} = 36$. In Fig. 6, simulation result of both BCDD and M-BCDD is presented.

As can be seen from Figs. 3 to 6, in all cases the proposed method outperforms the BCDD effectively, the BCDD codes, are the best existing SFBCs in the literature to the best of author's knowledge.

For example, from Fig. 1, at a bit error rate (BER) $= 10^{-5}$ and for the channel with delay spread of 5 µs, the M-BCDD achieve about 2 dB gains over the BCDD codes. Here, it is also helpful to present the CA values for BCDD and M-BCDD to back up the simulation results by confirming the derivatives of Section 4. Since the channels are relatively quasi-static, the CAs are very low, therefore, ratio of BCDD's CA to M-BCDD's CA are tabulated in Table 1.

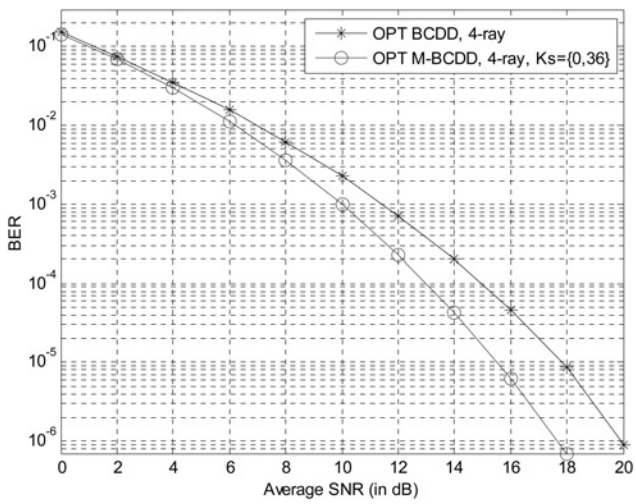

Fig. 6 *BER against SNR for the 4-ray equal power channel model with delay spread [0, 5, 10, 15] µs, PCSI*

Table 1 CAs of BCDD and M-BCDD

	No CSI – 5 µs	No CSI – 1 µs	PCSI – 2 ray	PCSI – 4 ray
$\frac{CA_{M-BCDD}}{CA_{BCDD}} \simeq$	12	25	11	19

From Table 1, it can be seen that the values are in agreement with the simulation results of Figs. 3–6. As expected from the results obtained in Section 4, the CA of the M-BCDD is more than ten times greater than that of BCDD's. Therefore, it is confirmed the proposed permutation scheme is capable of offering a higher CA value and therefore it gives superior performance compared with latest STFBC solutions. In addition, one can easily observe that the BER curves of both BCDD and M-BCDD have similar slopes at high SNRs, which means, as was discussed earlier, the proposed method does not change the diversity order.

8 Conclusion

In this paper, we established that published permutation methods for maximising the CA of STFBCs could not satisfactorily fulfil the requirements when channel response was correlated for different time slots. To overcome this challenge, we introduced a novel permutation scheme which exploited both temporal and frequency correlation matrices. Theoretical analyses are presented to demonstrate the superiority of proposed method to the published ones. The proposed method also particularly used to form a modification of BCDD codes. Finally, simulation results are performed to confirm performance improvement of modified BCDD over the conventional structure of BCDD codes.

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